NUMERICAL METHODS FOR DETERMINING THE OUTER LIMIT OF THE JURIDICAL CONTINENTAL SHELF OF A COASTAL STATE

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ABSTRACT

Bathymetric data sets used to compute the maximum change of gradient at the foot of the continental shelf usually require pre-processing operations such as resampling and numerical filtering. These operations may severely distort the shape of original data.

This paper proposes an alternative method that avoids uncertainties in the establishment of outer limits. The goal is to take a symbolic approach to the problem that consists of fitting functions to two- or three-dimensional models of the continental shelf. The application of symbolic functions, strictly functionals, has advantages over the use of vectors of numerical data. Functionals may be composed of a set of analytical functions that are at least twice differentiable, and which resemble as closely as possible the shape of the continental shelf in two- or three-dimensions. The correlation of functionals with the continental shelf model may be precisely quantified by statistical methods.

Once the functional parameters are established, analytical derivatives that take into account the elected coordinate system may be calculated to determine the maximum of the second derivative, i.e. the maximum change of gradient. This solution is unique. Featuring comparable levels of variability between a given series of data points and their derived functional, this procedure qualifies as a "well posed method". Among the strengths of the method proposed is that it requires neither data resampling nor filtering, thereby ensuring that bathymetric observations remain unchanged during computation. The described algorithm yields a unique solution to the problem of finding the maximum change of the gradient at the foot of the continental slope.

Introduction

Several methods have been proposed for implementing UNCLOS Article 76. The article establishes several methods for determining the outer limit of the juridical continental shelf of a coastal state: Distance Formula, Sediment Thickness Formula, 2500 Mts. isobath plus 100nm., etc.

This paper is focused on the Distance Formula, which projects the foot of the slope, FOS, of the continental shelf 60 nm. seaward. Its goal is to propose a method which:

- finds the FOS without uncertainties and
- evaluates quantitative measures of the error of every calculation step.

The algorithms presented here were first tested using Sandwell and Smith bathymetric data for some continental shelves with different morphologies and finally tuned with bathymetric observations from the Uruguayan continental shelf.

Motivation

UNCLOS Article 76 defines the Foot of The Continental Shelf as the maximum change of gradient at its base.

Numerical treatment of bathymetric data by smoothing or filtering procedures, as the Article 76 mentions, requires pre-processing operations which may distort the original data of hydrographic campaigns.

Prior of filtering using Fourier Transforms data should be resampled. Election of wave numbers for those filters must be carefully studied. Usually perturbations, which make unfeasible numerical derivatives of the continental shelf, contain spectral components very similar to those of the continental shelf itself. Trying to filter the perturbations may severely distort its shape. Varying the wave number of the filters makes the maximum gradient variation point ambiguous.

Others methods of filtering were tested, like those based on Minkowsky Algebra for erosion and dilatation of the continental shelf. The results were quite good, but the second goal: evaluate quantitative measures of the error of every calculation step, was not fulfilled.

Proposed Method

Instead of a pure numerical treatment of the problem a symbolic one can be used. Functionals can be fitted to the given numerical data and analytical derivatives could be found.

This method has several advantages: data needs neither to be resampled, nor filtered, it is not altered during calculation, bathymetric values are those obtained during the hydrographic campaign "as they are" and statistical values of errors can be precisely obtained.

The continental shelf can be described by a function f of latitude and longitude over depth. So if the function is differentiable, its directional derivative along $\boldsymbol{v}$ (the direction of the ship's course) written in terms of the gradient ∇f is:

$$Df(\vec{x})\vec{v} = \nabla f(\vec{x}) \cdot \vec{v} = \left[\frac{\partial f}{\partial x}(\vec{x})\right]v_x + \left[\frac{\partial f}{\partial y}(\vec{x})\right]v_y \qquad (1)$$

It should be pointed that partial derivatives of $f(\mathbf{x})$ for obtaining its gradient can not be evaluated, instead of it, given $f(\mathbf{x})$, the left side of equation (1) will be calculated.

Vector $\boldsymbol{v}$ usually is assumed unitary. If α were a real positive number, $\alpha\boldsymbol{v}$ is collinear to $\boldsymbol{v}$. If $\boldsymbol{v}$ is replaced by $\alpha\boldsymbol{v}$ in (1), $\nabla f(\mathbf{x}).\alpha\boldsymbol{v} = \alpha[\nabla f(\mathbf{x}).\ \boldsymbol{v}]$, so the directional derivative doesn't depend just on the point $\mathbf{x}$ and the direction $\boldsymbol{v}$. A unitary vector $\boldsymbol{v}$ was used. What is actually done by the choice of α is to define a scale, in this case the scale is Nautical Miles.

Hydrographic ships tracks may be straight lines, Article 76 suggests that they should be as perpendicular to isobaths as possible, in this case one of the members of the right side of equation (1) "vanishes", making calculations easier. In fact, using the chain rule, if the track of the ship were $\sigma(p)$,

$$\frac{d}{dp} f(\sigma(p)) = \nabla f(\sigma(p)) \cdot \sigma'(p) \qquad (2)$$

which is the directional derivative along the direction of $\sigma'(p)$. So it is not necessary that the ship's track $\sigma(p)$ be a straight line. It is enough to say that $\sigma(p)$ is known and that $\sigma'(p)$ is parallel to $\nabla f(p)$.

The key of the problem is to find $f(\mathbf{x})$, a function which resembles in two or three-dimensions the shape of the continental shelf. As real numbers and a space of functions are both Banach (normed and complete) vector spaces. Variational calculus enables all the considerations above mentioned to be applied to functionals, i.e. a special class of functions where its arguments are functions as well. In the 2D case, the functional should fit data of the form [distance – depth], this data can be extracted from the actual one [latitude – longitude – depth] using the proper spherical geometry, WGS-84 was used. Choosing smooth, infinitely differentiable, or at least twice differentiable functions as functional arguments, the problem can be solved.

A set of sigmoidal and exponential forms was selected. The criteria for fitting this functional to the available data was to minimize the value of:

$$\chi^2(\vec{a}) = \sum_{i=1}^{N}\left[\frac{d_i - f(p_i, \vec{a})}{\sigma_i}\right]^2 \qquad (3)$$

The chi-square function of a set of values ***a*** (the values of all the parameters involved in the functions used), equals the summatory over *N* points (the points of data) of the square of: the difference of the bathymetric observation d_i and the value of the function *f* at point p_i with parameters ***a***. σ_i is the standard deviation of the measuring device, so the chi-square value will be less affected if this value is high. So, given a functional of a set of functions, the goal is to find the parameters ***a*** which minimize the chi-square, thus maximizing the likelihood. For accomplishing this the Levenberg-Marquardt algorithm was used. This method varies smoothly between the steepest descent and the Hessian matrix-based algorithms over the error surface.

2-D Application

The first step for this calculation was to convert the original observations in the form [latitude-longitude-depth] to the elected scale, (nautical miles in this case) resulting in [distance-depth] according to WGS-84 spherical geometry. Distances are considered from the track starting point.

The functional chosen, actually an “ansatz” was:

$$p(d) = d_m + \frac{d_p}{1 + e^{-D_s(d-d_o)}} + \sum_i A_i e^{-D_i(d-d_i)^2} \qquad (4)$$

The parameters vector ***a*** = [d_m, d_p, D_s, d_o, A_1, D_1, d_1, A_2, D_2, d_2, A_3, D_3, d_3, ...] which minimizes the value of chi-square must be found. Once it is known, it can be used in the second derivative of equation (4) for calculating the gradient variation.

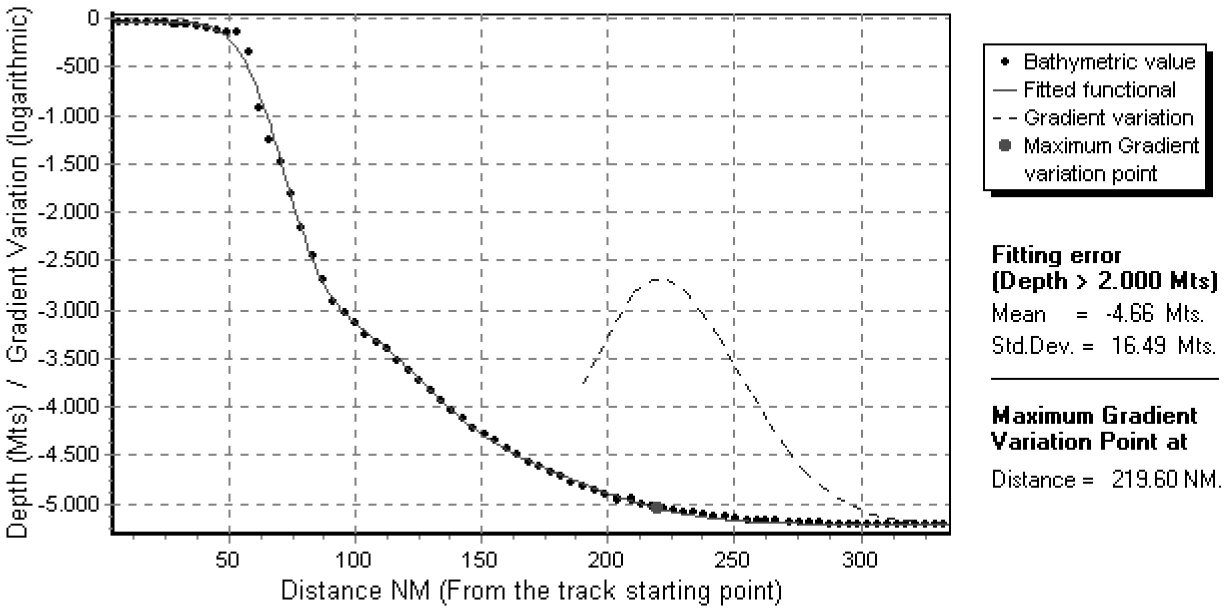

Figure 1
(Bathymetric values shown are the minimum as to make the figure clear.)

The second directional derivative of equation (4) is:

$$D^2 p(d) = 2\frac{d_p D_s^2 e^{[-D_s(d-d_0)]^2}}{\left[1+e^{-D_s(d-d_0)}\right]^3} - \frac{d_p D_s^2 e^{-D_s(d-d_0)}}{\left[1+e^{-D_s(d-d_0)}\right]^2} + 2\sum_i A_i D_i e^{-D_i(d-d_i)^2}\left[2D_i(d-d_i)^2-1\right] \quad (5)$$

In Figure 1 the bathymetric values, the fitted functional, the gradient variation and the maximum gradient variation point are shown. The fitting error was quantified. The mean error is –4.66 Mts., and the standard deviation is 16.49 Mts. This means that $p(d) \pm 16.49$ Mts. contains 68% of the bathymetric observations, and approximately 95% of them are within $p(d) \pm 33$ Mts.

3-D Application

The methods used for 2-D applications can be extended to 3-D models. In this case, resembling the continental shelf shape by a functional is more difficult. The difficulties lie in the amount of parameters involved in that functional.

Figure 2 shows the Uruguayan continental shelf, it was built with nearly 25.000 observed bathymetric values. The surface seems to be quite regular, so an adequate functional may be found quite easily. In countries where it is not, the continental shelf could be considered piecewise regular, "pieces" may be overlapped for obtaining a smooth FOS line.

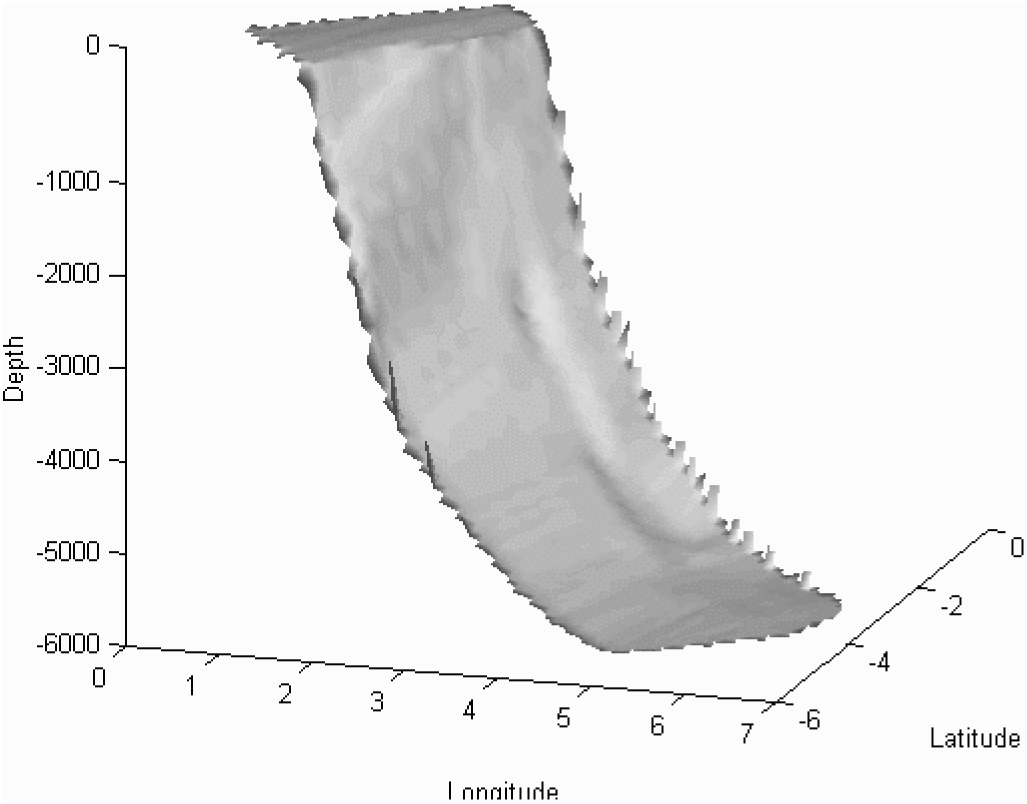

Figure 2

If

$$f_i(y) = A_i e^{-\frac{(y-y_i)^2}{d_i}}$$

Functionals like

$$p(x,y) = d_m + \frac{d_p}{1+e^{\frac{-x+f_o}{f_1}}} \qquad (6)$$

where exponential forms for adjusting the position and dispersion of a three-dimensions sigmoid were used successfully.

Statistical measures of the likelihood of the fitted function were not as good as in the two-dimension case. Anyway, the results were quite good, FOS in this model is within 10% of the one shown in Figure 1.

Calculus complexity grows as the square of the length of the parameter vector. For rescuing the perturbations of the continental shelf in a three-dimensions model, more functions should be considered.

Conclusions

Symbolic approaches to the problem of finding the foot of the slope of the continental shelf of a coastal state have some advantages over numerical treatments.

Filtering or smoothing are not necessary. Thus avoiding the corruptness of the observed bathymetric values. So, likelihood of the models used may be precisely established.

The foot of the slope point is calculated analytically, from the fitted model, and it is unique.

The proposed method maximizes the likelihood of the model, calculating the optimum parameters vector, with a least square error criterion. Robustness of the algorithm lies in the possibility of establishing the fitting error, assuring that the parameters vector is optimum and it does not correspond to a local minimum of the error surface.

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