"THE EFFECTS OF USING DIFFERENT ALGORITHMS FOR CALCULATING THE FOOT OF SLOPE BASED ON THE MAXIMUM CHANGE OF GRADIENT"

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Abstract

The extent of the outer limit of the continental shelf of a coastal state is greatly dependent on the placement of the foot of the continental slope. Both the Gardiner line (based on sediment thickness) and the distance formulae line (foot of slope + 60M), which are possible extensions to the continental shelf, are based on an interpretation of the foot of slope points.

UNCLOS Article 76 defines the foot of slope as: "4. (b) In the absence of evidence to the contrary, the foot of the continental slope shall be determined as the point of maximum change in the gradient at its base."

This article discusses the maximum change of gradient of bathymetric profiles and why this point may not be as unique in a real case, as it may seem from a theoretical point of view. Different algorithms for determining the maximum change of gradient will be presented. Also we will see how the choice of algorithm and the algorithm's parameter settings affects the placement of the foot of slope point.

The article describes some common pitfalls one should avoid when computing the change of gradient in a real case, along with a new approach for calculating the maximum change of gradient. This algorithm gives good and stable results in most cases.

The Geocap software package with its bathymetric profile analysis tool is used to examine the different algorithms.
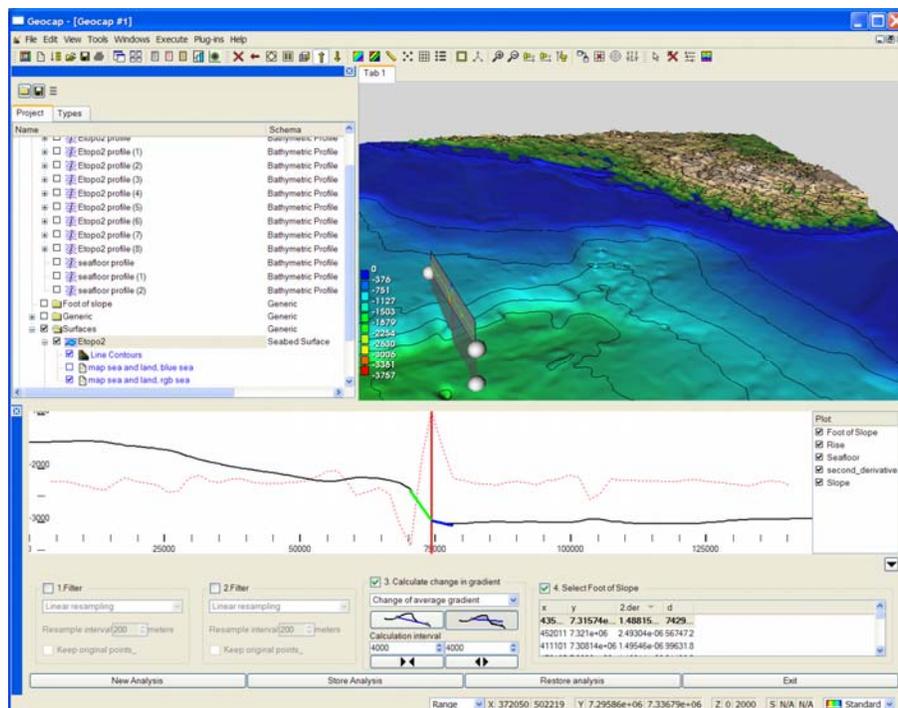
1   Introduction

This paper gives an overview of some of the aspects one should consider when selecting a foot of slope point based on the maximum change of gradient criteria. Other papers presented at previous ABLOS conferences discuss similar issues. Stagpoole et al.[2] and Alcock et al.[3] present methods of identifying foot of slope points with no special focus on how they computed the second derivative. Carrera [4] presents a more technical approach to the problem discussing different filtering algorithms on surface grids.

This paper has a technical approach to finding the maximum change of gradient, and discusses different algorithms for such computations on bathymetric profiles. Even if the paper may be technically oriented, the mathematics in this paper is quite simple, and you should be able to read it even with limited mathematical skills.

1.1   The maximum change of gradient

The gradient is a mathematical term, connected to functions in any dimensions. The seafloor surface can be represented mathematically as a function in two dimensions. The gradient of a point on the surface is a vector which points in the direction where the surface is steepest. The length of the vector corresponds to the steepness of the surface at the point.

On a bathymetric profile, the gradient is the same as the derivative of the function. The change of gradient can therefore be interpreted as the second derivative of a function. Or in other words: the maximum change of gradient is the point where the function has the largest curvature, or where it bends the most.

1.2   What is the problem?

The derivative of a function is a mathematical term related to continuous functions. In practice, this is never the case. A bathymetric profile consists of discrete depth measurements. In order to compute the change of gradient, algorithms can be used, which approximates the second derivative.

When examining the change of gradient, for placing the foot of the continental slope, one is often not interested in every local variation in the profile, but rather the change of a tendency, over a larger interval.

This is supported by the CLCS Scientific and Technical Guidelines [1] which state

"5.3.1. The Commission recognizes that filtering and smoothing of bathymetric data might be required in order to facilitate the identification of the location of the foot of the continental slope at the point of maximum change in the gradient at its base. This procedure might be required in some instances because the use of second derivatives of the bathymetric surface produces an enhancement of all features which may obscure the exact location of the foot of the slope."

And

"5.3.2. Filtering in signal theory presupposes a clear differentiation between signal and noise, that is, what is to be regarded as wanted and unwanted information. In the context of the application of paragraph 4 (b), the shelf, the slope and the rise are signal. Any other information that obstructs the location of these features is regarded as noise."

We will see that the choice of algorithm and its parameter settings may produce different results with different maximum points.

In the simple case, the bathymetric profile has a marked change in steepness between the slope and the rise. The maximum change of gradient is very obvious, and most algorithms would produce the same result.
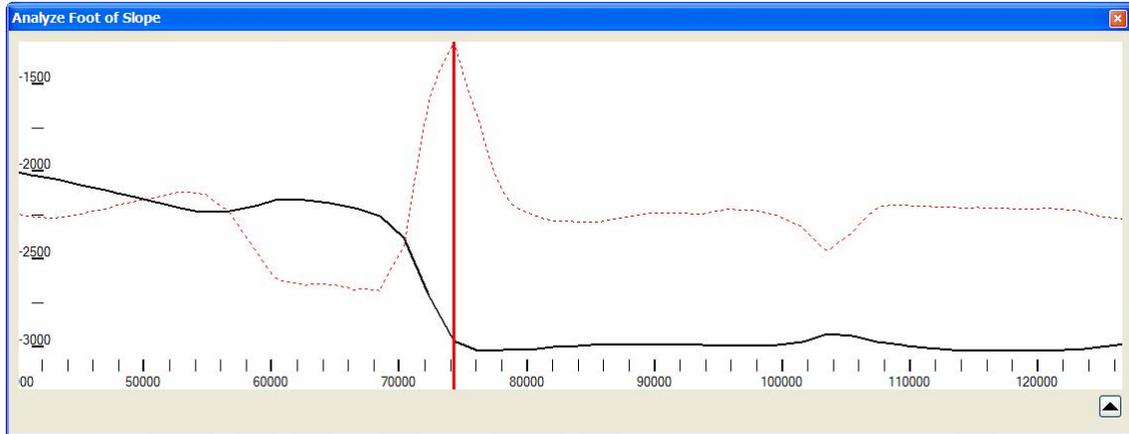
However in other situations, the transition between the slope and the rise is more gradual, and local second derivative from small variations in the profile may exceed the second derivative in the transition between the slope and the rise. An example is visualized in the next figure.
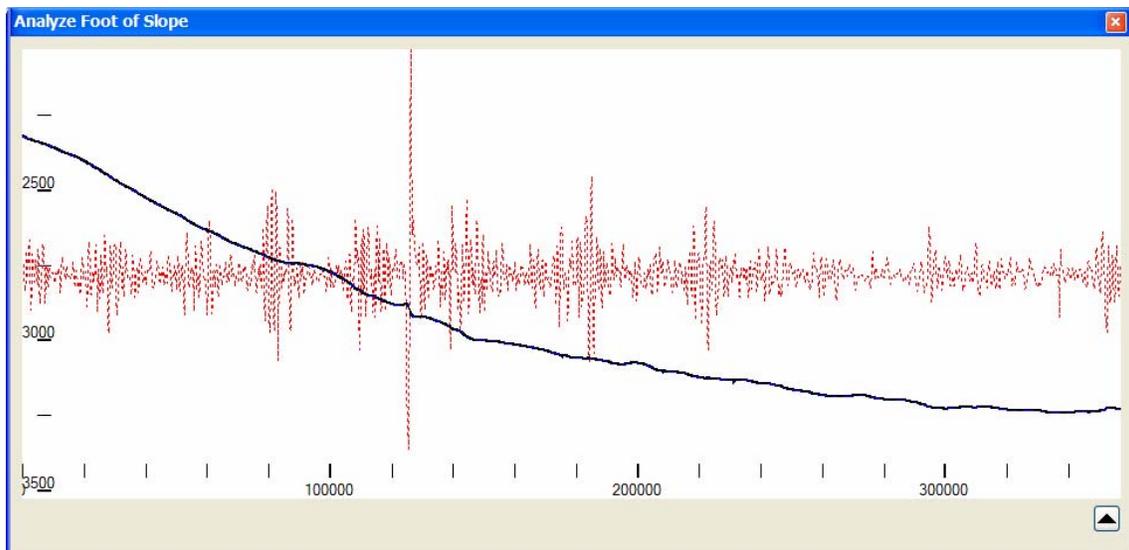
A simple finite differences method is used in order to compute the second derivative. In the profile, it is not easy to say where the rise ends, and the slope begins. The maximum change in gradient is triggered by a small but sudden change in gradient. We will study this profile in chapter 5, and try to find a stable result for the maximum change of gradient. First we will present some algorithms which may be used on the profile in order to find the maximum change of gradient.

2   Algorithms used when computing the change of gradient

The algorithms described in this chapter are available in the Geocap foot of slope analysis tool. Most of these are common and well known algorithms, and only a short description is given.

2.1   Re-sampling

The bathymetric profile is re sampled using a specified interval. The new points are linearly interpolated, and placed on the straight line between the points on the original line. If the new sampling rate is denser than the original line, the several points may be positioned on the same straight line, creating a second derivative equal to zero.

2.2   Spline

A variation diminishing spline approximation may also be used on the input points. The algorithm uses a cubic spline. The result is a piecewise polynomial with continuous first and second derivative, which approximates the input points. The spline is re sampled using the specified interval. This algorithm will give a better result than linear re-sampling. If more points are introduces between two input points they will not necessarily be placed on the straight line between the input points, creating a zero second derivative, as they would when using linear re sampling.

2.3   Gliding average

The gliding average algorithm sets the value of each point to the average of itself and its neighbouring points. The number of neighbouring points used in the calculation can be specified as a parameter to the algorithm.

2.4   Median filter

The median filter is similar to the gliding average filter. The value of each point is set to the median value of itself and its neighbouring points. The number of points used in the calculation can be specified as a parameter to the algorithm.

2.5   Douglas Peucker

The Douglas Peucker algorithm creates a new line similar to the original input line, with the maximum deviance from the original line less than a given threshold. The Algorithm is an iterative process starting with a single line from the first to the last point of the original line. The point with the greatest deviance is inserted in the new line, until the distance from all the points on the bathymetric profile to the new polygon fulfils the threshold criteria.

One should be aware when using this algorithm that the resulting line does not necessarily represent the trend of the input points, but rather strives to minimize the distance from the new line to the extreme values of the input line.

2.6   Fourier low pass filter

A Fourier low pass filter is common in signal theory. A Fourier transform approximates the input profile with a sum of sine and cosine functions with different frequencies. When using a Fourier low pass filter, only the sine and cosine parts with a lower frequency than the cut off frequency are used in the approximation. The result is a smoothened version of the input profile. When the cut of frequency is raised, more details are introduced in the output profile.

3   Computing the change of gradient

In this chapter we introduce two methods for calculating the change of gradient. The first algorithm is quite commonly known. The second one is introduced with Geocap, and is described in detail. We call it the "Geocap change of average gradient" method

3.1   Finite differences

When approximating the derivative using finite differences, the algorithm is similar to the definition of the derivative except that because the function is not continuous, the limit value which tends towards zero is exchanged with a finite difference.

$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \right)$$

is replaced with

$$f'(x_i) \approx \left(\frac{\Delta y_i}{\Delta x_i} \right) \text{ where } \Delta x_i = x_{i+1} - x_i \text{ and } \Delta y_i = y_{i+1} - y_i$$

This is called the forward difference method. A similar method which involves x_i and x_{i-1} is called the backward difference method. The second derivative may be calculated by using the result of one method as input to the other.

It can be proved by using Taylor's Theorem that this approximation gives good results if the input points are regularly sampled ($\Delta x_i = \Delta x_{i-1}$)

It is important to note that the derivative at the point x_i only involves the value at x_i and its two neighbouring points. The algorithm is therefore very sensitive local variations and noise. This is why a variant which uses more input points and examines the average steepness over a larger interval has been developed.

## 3.2   Geocap Change of Average Gradient

The change of average gradient was developed at Geocap when we saw the need for a method which works on a larger interval than the finite differences method.

In order to compute the change of gradient at a point, the method uses the average gradient in a given interval before and after the point, and compare the two to find the change of gradient in that point.

The algorithm can use two different methods for computing the average derivative at an interval the two methods are explained here:

Method I

The bathymetric profile is given by the points $\{p_0, p_1 \dots p_n\}$. Let f(x) be the piecewise linear polynomial which interpolates the point $\{p_0, p_1 \dots p_n\}$.
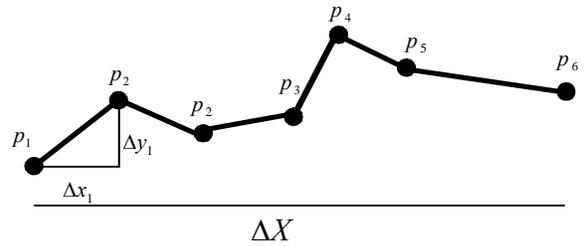
Let $p_i = (x_i, y_i)$ and $\Delta x_i = (x_{i+1} - x_i)$ and

$$\Delta y_i = (y_{i+1} - y_i), \text{ and } \Delta X = \sum_{j=0}^{m} \Delta x_{i+j} = x_{i+m} - x_i$$

We know that because f is piecewise linear $f'(x) = \dfrac{\Delta y_i}{\Delta x_i}$, when $x \in (x_i, x_{i+1})$, lets call this

derivative for f'_i

The average gradient of f between x_i and x_{i+m} can be computed by summing up the derivatives in the interval, and weighting them by the length of the respective interval, and dividing the sum by the length of the total interval. That is:

$$\overline{f'(x)} = \frac{1}{\Delta X} \sum_{j=0}^{m} \Delta x_i f'_i$$

by substituting f'_i with its definition, we find that:

$$\overline{f'(x)} = \frac{1}{\Delta X} \sum_{j=0}^{m} \Delta x_i f'(x)_i = \frac{1}{\Delta X} \sum_{j=0}^{m} \Delta x_i \frac{\Delta y_i}{\Delta x_i} = \frac{\sum_{j=0}^{m} \Delta y_i}{\Delta X} = \frac{\Delta Y}{\Delta X} \text{ where } \Delta Y = \sum_{j=0}^{m} \Delta y_{i+j} = y_{i+m} - y_i$$

We are left with:

$$\overline{f'(x)} = \frac{\Delta Y}{\Delta X}$$

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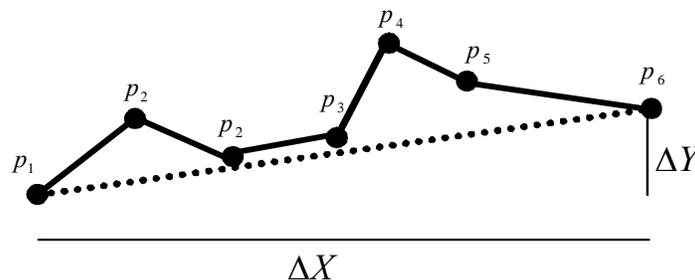
Method II

Another algorithm for computing the average gradient in an interval has also been developed. Let the bathymetric profile be represented by the function $f(x)$. $f'(x_j)$ is computed by studying the derivative of a linear polynomial $p(x)$ where:

$$p(x_j) = f(x_j) \text{ and } \int_{x_i}^{x_k} p(x) = \int_{x_i}^{x_k} f(x) \text{ where}$$

$$x_i \le x_j \le x_k \quad (x_i \ne x_k)$$
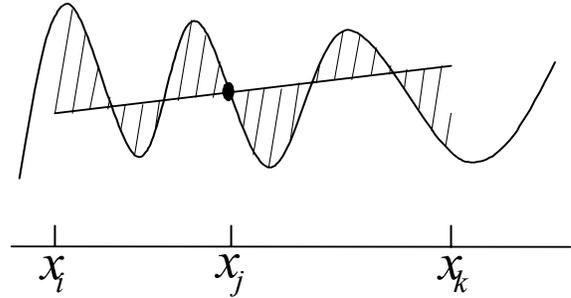
In other words, the area between p and f above p is equal to the area between p and f below p.

3.2.1 Computing the change of gradient

With both methods for finding the average gradient on an interval, the change of gradient is computed the same way. For each point on the input line, the average gradient is computed on an interval on each side of the point, and the change of gradient is computed by:

$$\Delta G_j = \frac{f'(x_j)^+ - f'(x_j)^-}{\Delta X^+ + \Delta X^-}$$
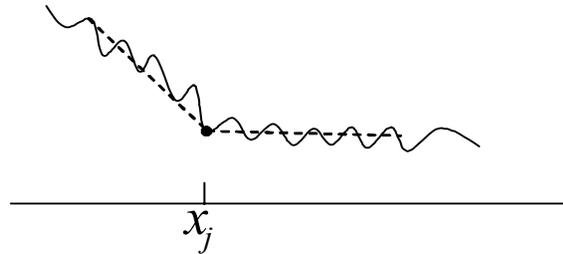
Where ΔX^+ is the length of the interval after x_j and ΔX^- is the length of the interval before x_j. $f'(x_j)^+$ is the average gradient in $(x_j, x_j + \Delta X^+)$ and $f'(x_j)^-$ is the average gradient in $(x_j - \Delta X^-, x_j)$


The results are similar to the second derivative of the input function, except that the algorithm finds changes in tendencies, rather than local variations. The sensitivity of the algorithm is adjusted by adjusting the size of the average gradient interval.

If the average gradient calculation interval is set to the sampling rate of the input points, the change of gradient is reduced to the finite difference method similar to the method described in 3.1

In the implementation of this method, we have added has the possibility to set the calculation interval on the left and right side of the calculation point individually. This makes the algorithm very flexible, because it can adapt to profiles with for example a long slope and short rise.

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4   Geocap's Bathymetric Profile Analysis tool.

Geocap has developed a bathymetric profile analysis tool for locating the foot of slope point. The dialog is built up of two parts. The upper part is used for visualizing the profile, the second derivative and the results of the input filters if they are used. The bottom part is used to apply settings to the foot of slope calculation. The bottom part may be minimized in order to give more space to the profile visualization.
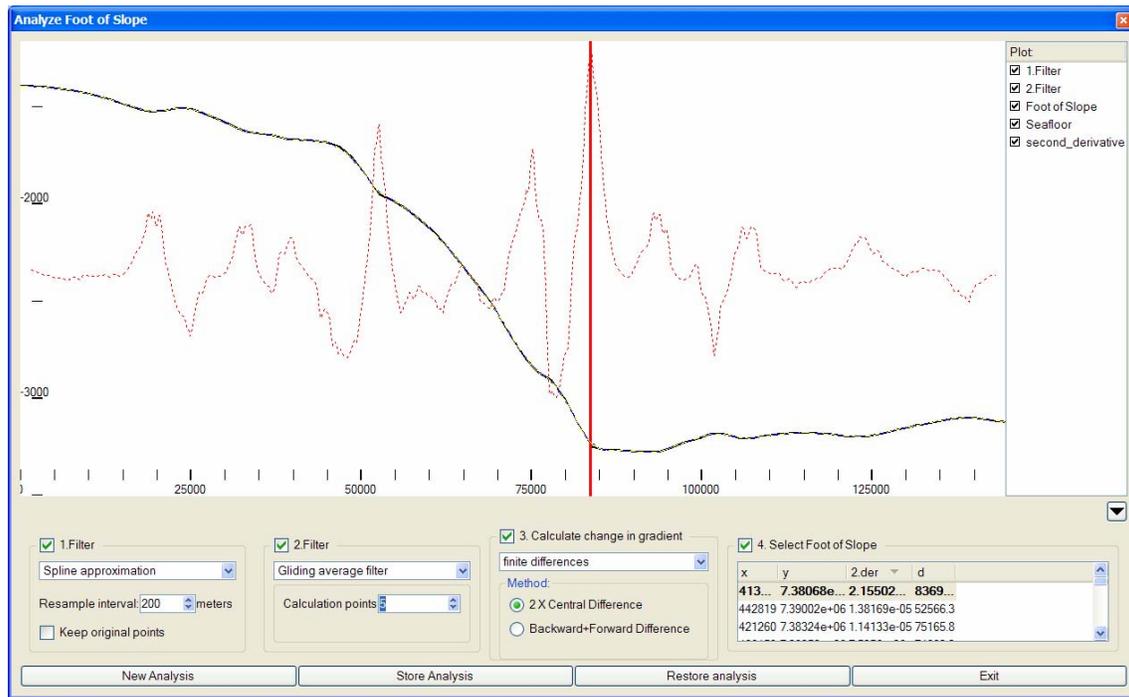
A bathymetric profile is used as input. The input can be filtered and modified through two optional filters, before the change of gradient calculation is preformed. This provides good flexibility.

All the algorithms described in chapter 2 are available as the first or second filter.

5 Finding the foot of slope.

In this chapter we will be using the Geocap Analysis tool to study the change of gradient on a specific profile. The profile has been sampled from a 200x200 meter grid which is based on multi beam echo sounder. This profile is very long. It is 360 km from beginning to end, and particularly difficult because it has no obvious foot of slope points.

5.1 Using finite difference methods

We start with the simplest method. The profile has been regularly sampled, and the method of finite differences is used. The result is presented in the figure bellow.
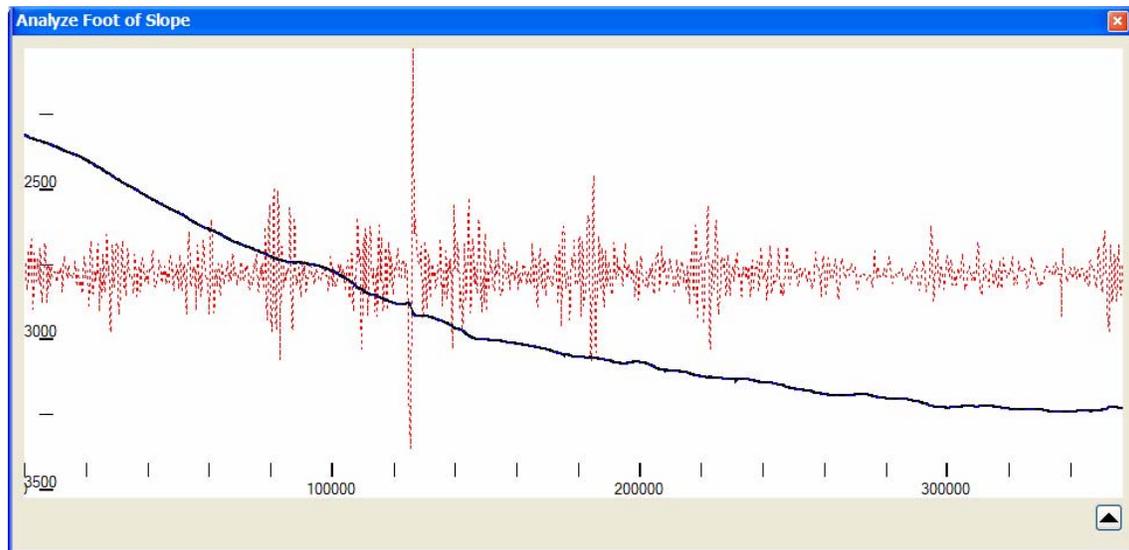

The result is not very pleasing. The change of gradient has great variations. The maximum change of gradient is placed at a small bump, which by examining of the original grid seems to be small a local variation on the seafloor. It seems like we need to smoothen the input data in order to find the foot of slope point.

5.2 Using gliding average

In order to smoothen out the local variations, a gliding average filter is used before the finite differences method is used to calculate the change in gradient. The result is presented on the next page.

The second derivative marked with a red dotted graph, does not seem to be smoothened. This is actually not the case, but the graph is scaled to fit the window, so as the second derivative gets smaller, so does the second derivative maximum. Relatively the gliding average does not seem to reduce the variation in the second derivative noticeably.

The results are not very pleasing. From the figures, we see that the maximum change in gradient moves down the slope, as the smoothing increases. It does not seem to fix at any point.

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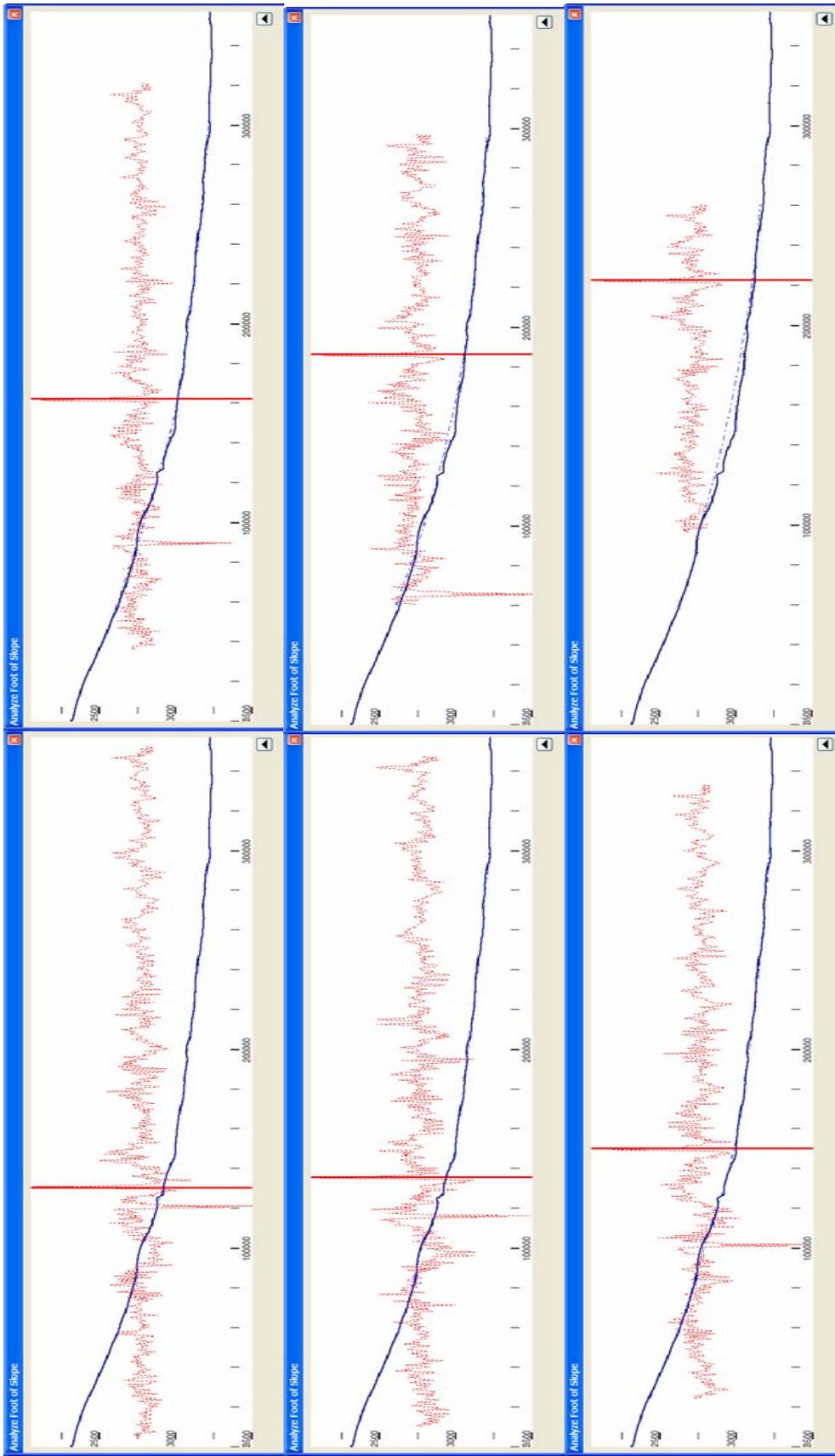
The number of points in the gliding average calculation increase from the top to the bottom and from the left to the right.

5.3 Douglas Peucker

The Douglas Peucker algorithm has been suggested as a method for simplifying the bathymetric profile in order to find the foot of slope point. This may give seemingly good results. The first points inserted using the algorithm are the points with the greatest deviance from a straight line between the endpoints of the profile. These points often cohere with points on the line which seem to be foot of slope points. But one should be careful when using this algorithm. The points selected in the Douglas Peucker algorithm are the points with the greatest deviance from the straight line, and this is not necessarily the point with the maximum change of gradient. The Douglas Peucker algorithm has been used on the profile in the screen shot bellow.
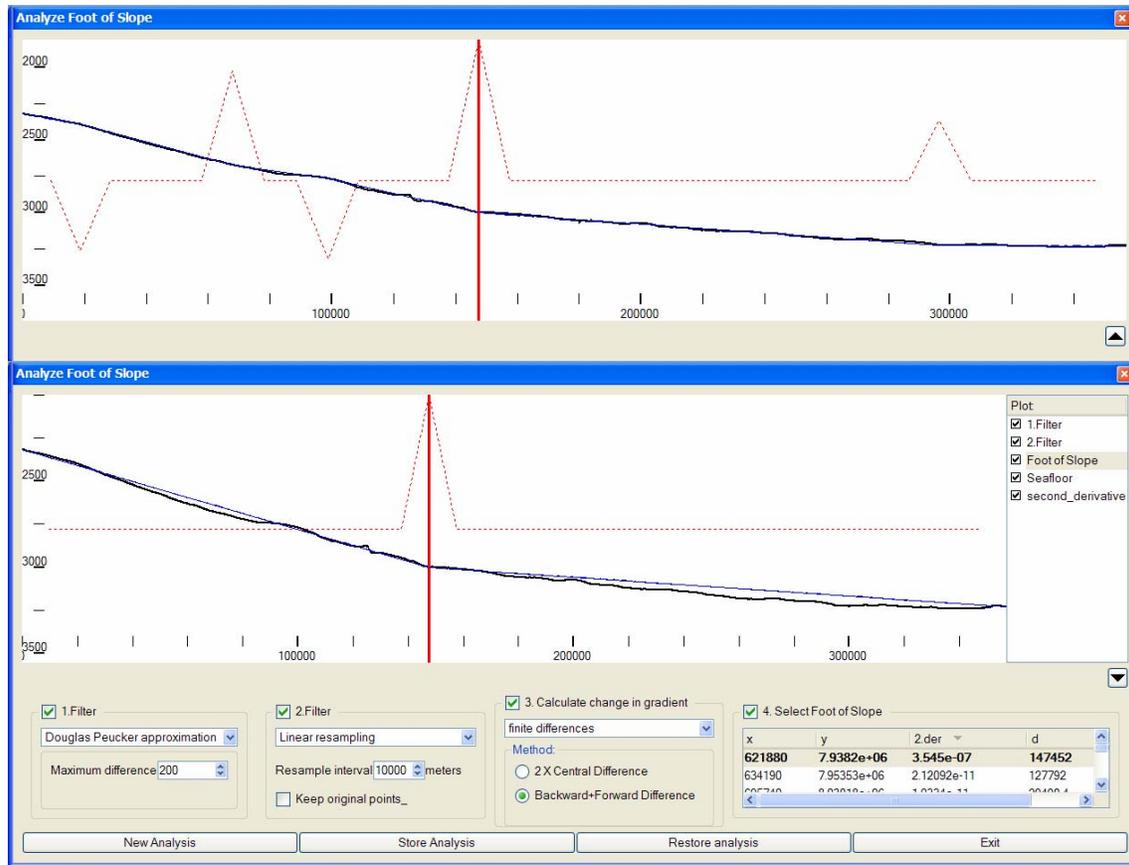
5.4   Fourier Low Pass Filter

The Fourier low pass filter has been used on the bathymetric profile with different cut-off values. The results are presented on the following page.

The results show that the maximum change of gradient varies as the cut of frequency changes. In the last figure, the change of gradient has the same maximum as the finite differences algorithm used directly on the input data. This is logical, as the Fourier filtered data approaches the input signal as the cut of frequency increases.

Oppose to the gliding average algorithm, the Fourier low pass filter seems to smoothen out the small variations in the input line, while keeping the general shape of the bathymetric profile.

The algorithm seems to give good results when using the correct cut off frequency. What the correct cut off frequency is, is however a matter of opinion, and open for discussion.
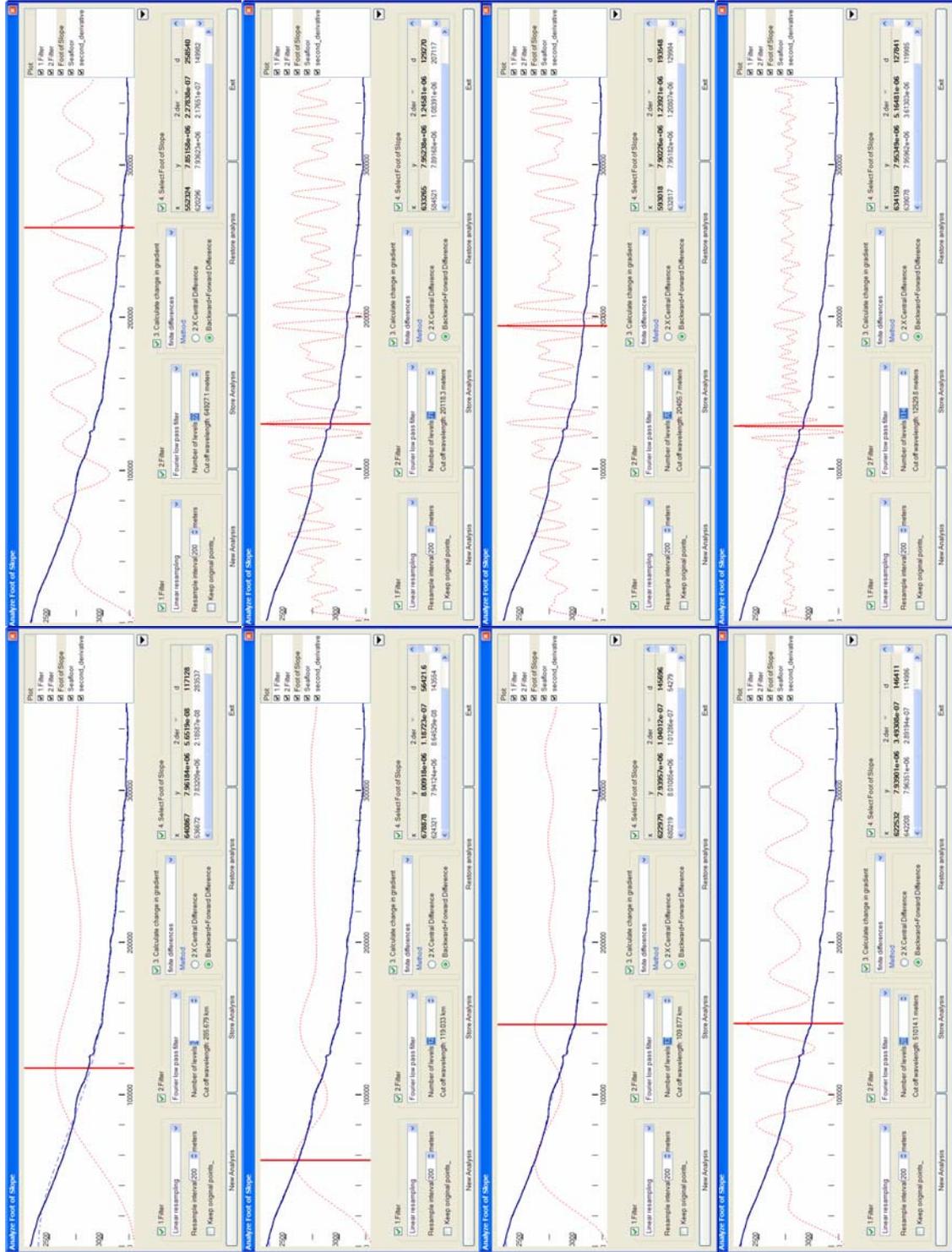
The figures cut off frequency increases from top to bottom and left to right. As the frequency increases, more details appear in the approximation. The details in the second derivative increases accordingly.

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5.5 Geocap Change of Average Gradient

The Geocap Change of Average Gradient algorithm was used on the profile. The result is shown in the figures bellow. From the figures we can see that a short interval gives a similar result as the finite differences method. This is as expected, because the algorithm reduces to the finite differences method when the average intervals tend toward the sampling interval.

As the interval increases, the variation in the second derivative diminishes. After a few steps, the maximum change of gradient fixes at a point. The maximum change of gradient is fixed at this point as the size of the interval increases.

In this example the algorithm uses the average gradient method II. The first method gives a similar result, and the same foot of slope point.

The average change of gradient was the only algorithm which seemed to stabilize at a single point. In the general case, the algorithm might not be this stable. The algorithm will only be stable as long as the slope and the rise have constant trends, and the computation interval is shorter than the slope and rise.

6   Ending remarks

The maximum change of gradient changed on this profile depending on the algorithm being used, and it can be argued that any of these points are valid foot of slope points. They are the maximum change of gradient using that particular algorithm and settings.

On this particular profile, the change of average gradient seems to give very stable results. The maximum change of gradient is fixed at one point as the calculation interval increases.

This profile is ideal for the change of average gradient as it has a very long slope and rise with a constant trend. When the interval increases, the slope and rise become more evident. The Geocap Change of Average Gradient method gives good and stable results in these cases.
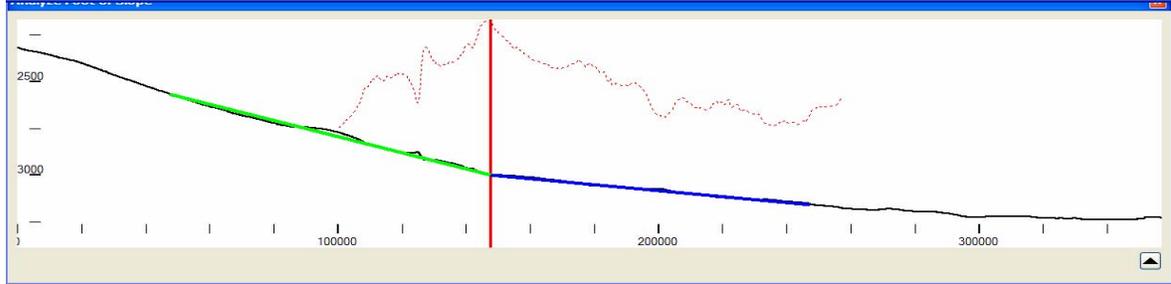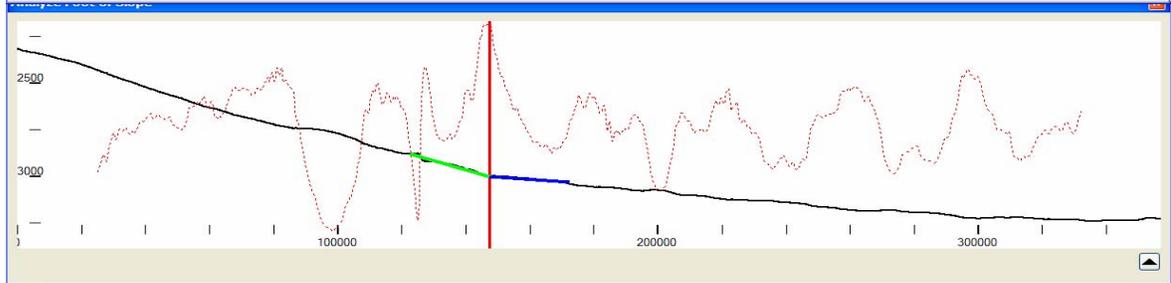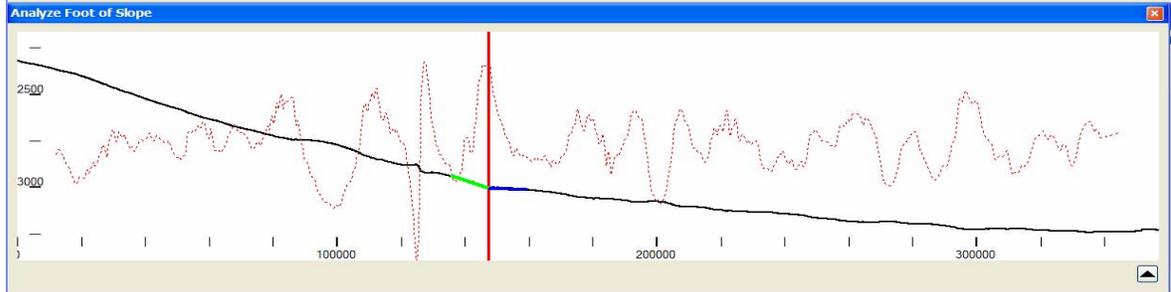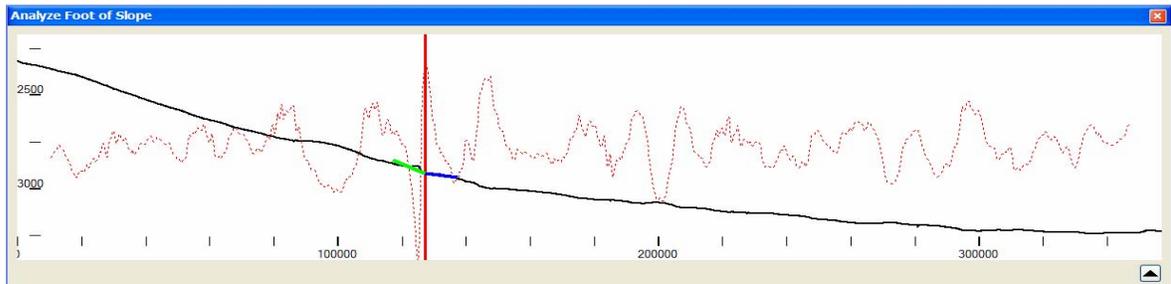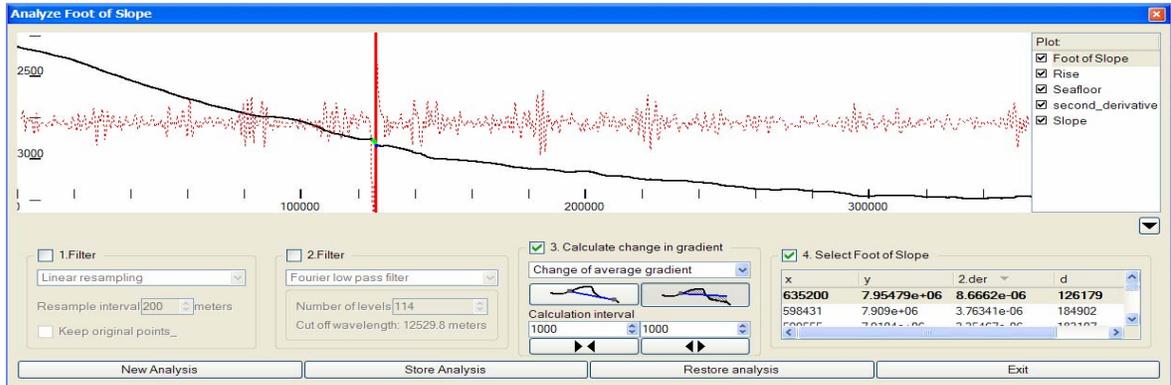
The size of the average gradient interval increases from the top to the bottom. The size of the interval and the average steepness is visualized with a blue and green line on each side of the maximum.

About Geocap

Geocap is a program which original purpose was to assist geo scientists in the oil industry. Modifications where made to the program in order to use it for UNCLOS Article 76 related work, and was presented by Egeland at the ABLOS conference in 2003 in [5]. The UNCLOS module has been further developed, and it has now many useful features for Article 76 related work. Geocap provides a modern and flexible working environment with 3D visualization.

In addition to an excellent foot of slope analysis, Geocap has methods for finding displacement lines such as the 200M line, 350M line and so on following the earth curvature, as well as methods for finding the 2500 meter isobath. Sediment thickness calculations can also be done in Geocap. These calculations can be based on imported sediment thickness points, or based on seismic data, which can be interpreted in Geocap, and depth converted based on stacking velocities.

Geocap also provides a lot of other functionality which is not specific to Article 76, such as gridding, plotting, volume calculations, geodetic conversions etc.

For more information about Geocap, please contact us at
home@geocap.no
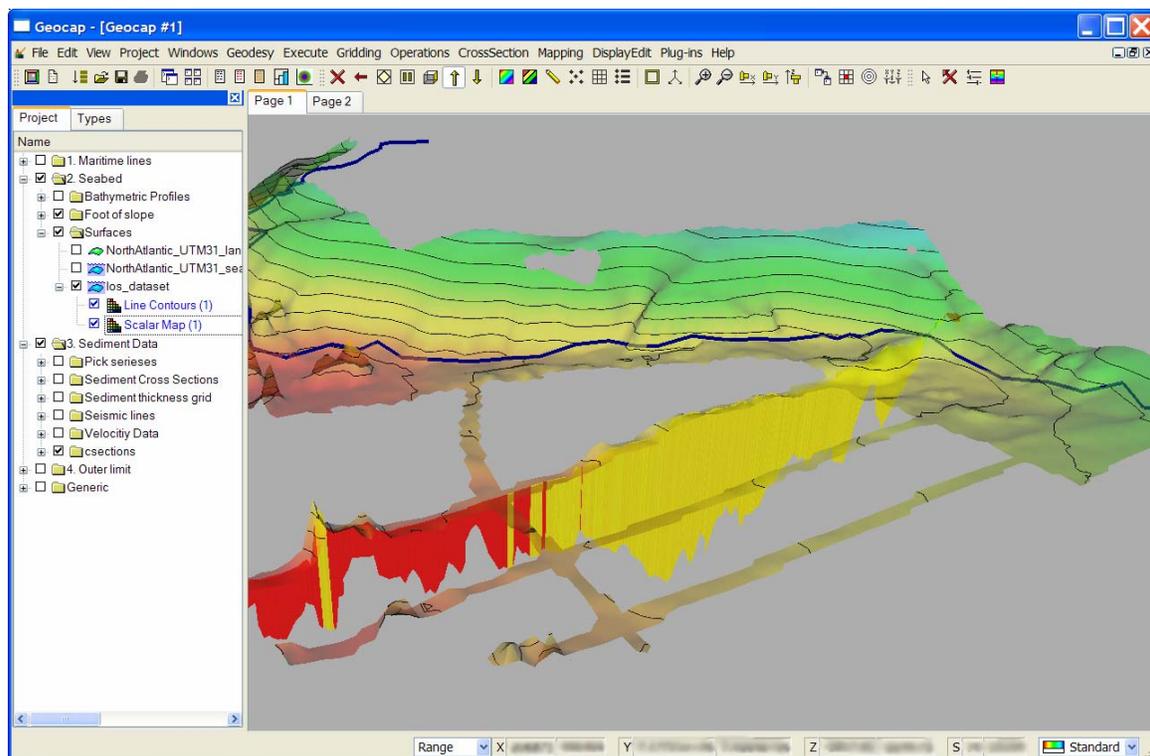
A visualization of a sediment thickness profile, generated from a seismic 2D line, interpreted and depth converted in Geocap. The yellow parts are thicker than 1% of the distance to the foot of slope. The foot of slope line is visualized as a thick dark blue line.

7   References

[1] CLCS/11, 1999. Scientific and Technical Guidelines of the commission on the Limits of the Continental Shelf. Commission on the Limits of the Continental Shelf.

[2] Stagpoole, V., Wood, R., Herzer R., Wright, I., Davy, B., Barnes, P. "Foot of the Continental slope in article 76" ABLOS '03

[3] Alcock, M.B., Colwell J.B, Stagg H. M.J. "A systematic approach to the identification of the foot of the continental slope – Article 76 UNCLOS" ABLOS '03

[4] Carrera G. "The Impact of the Seabed Roughness on the Location of the 2,500m Isobath and the Foot of the Continental Shelf" (PDF): ABLOS '99

[5] Egeland, O. "A new approach to mapping of the continental shelf" ABLOS '03