Vertical Reference Framework – Submitted by the IHB

IAG – International Association of Geodesy

The IAG inter-commission project ICP1.2 on vertical reference frames was tasked to prepare a proposal for the definition and realization of a global vertical reference frame (World Height System – WHS). The IHB has not been able to attend any further meetings of ICP1.2 since IHOTC7. ICP1.2 has finalised the report which will go to the IAG Congress and a copy is attached to this paper. It is not known at this point whether the project will be re-established by the Congress.

FIG – International Federation of Surveyors

As indicated at IHOTC7 the FIG Congress was held in Munich in October 2006. Captain Hugo Gorziglia and Steve Shipman attended on behalf of the IHB. Andrew Leyzack from Canada was elected as the Chair of Commission 4 (Hydrography) and the excellent liaison established with his predecessor, Adam Greenland, is continuing. The "FIG Guide on the Establishment of a Vertical Reference Surface for Hydrography" prepared by a WG of Commission 4 led by Ruth Adams from the UKHO was published as FIG Publication No.37 and presented at the Congress. This publication can be downloaded free of charge from the IHO website www.iho.int > INT Organizations > FIG.

IAG Inter-Commission Project
ICP1.2 Vertical Reference Frames

Final Report of IAG ICP1.2

(Jointly by Commissions 1 and 2)

For the period 2003 - 2007

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Terms of Reference of ICP1.2

The Earth's surface may be described by its geometry and the potential of the Earth's gravity field. The determination of heights includes both of these aspects - the geometric part and the geopotential part. Presently, space geodetic techniques allow an accuracy in geometric positioning of about 10-9 of the Earth´s radius in global and continental scales. Gravity field parameters, including the physical height components, can at present be determined only 2 to 3 orders of magnitude less accurately than the geometric parameters. Moreover, the current height reference frames around the world differ in their vertical datums (e.g., the mean sea-level at the fundamental tide gauges) and in the theoretical foundations of the height systems. There is no global height reference system defined or realized, as with the International Terrestrial Reference System (ITRS). Considerable progress in the definition and realization of an unified, global vertical reference system will be achieved from the data of the new satellite gravity field and altimetry missions.

Based on the classical and modern observations, the ICP1.2 on Vertical Reference Frames shall study the consistent modelling of both, geometric and gravimetric parameters, and provide the fundamentals for the installation of a unified global vertical reference frame.

Objectives

- To elaborate a proposal for the definition and realization of a global vertical reference system (World Height System – WHS);
- To derive transformation parameters between regional vertical reference frames;
- To establish an information system describing the various regional vertical reference frames and their relation to a world height frame (WHF).

Program of Activities

- Harmonization of globally used height data sets;
- Study of combination procedures for height data sets from different techniques;
- Study of information on regional vertical systems and their relations to a global vertical reference system for practical applications;
- Unification of regional (continental) height systems.

Members

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Meetings and workshops

– European Vertical Reference System Workshop, 5-6 April 2004 in Frankfurt on Main, Germany (draft minutes)

– Business Meeting of ICP1.2, 31 August 2004, on GGSM2004 in Porto, Portugal (minutes)

– Business Meeting of ICP1.2, 22 August 2005, on the IAG Scientific Symposium in Cairns, Australia (minutes)

– ICP 1.2 Workshop, 11-12 April 2006 in Prague, Czech Republic (minutes)

– ICP1.2 Splinter/Business Meeting, 28 August 2006 at the 1st IGFS Symposium in Istanbul, Turkey

Status and Results

The results of the work of the Inter-commission Project 1.2 are documented in **Conventions for the Definition and Realization of a Conventional Vertical Reference System (CVRS)** - File VRS_conventions_3.0_2007-05-01. In the CVRS conventions a general concept for the definition and realization of a unified, global vertical reference system is described. The CVRS conventions are aligned to the IERS 2003 Conventions. Parts of the IERS 2003 conventions are the basis for the CVRS conventions.

A global unified vertical reference system for an International Vertical Reference System (IVRS) can be realized by:
– A global network of stations with coordinates in ITRF and geopotential numbers referred to a conventional global reference level. This network should include co-location of permanent GNSS, tide gauges, permanent (SG) and periodical (AG) gravity stations.
– A global reference level derived from a conventional global gravity model (CGGM) from satellite gravity missions only in combination with a global sea level model from satellite altimetry.
– Both based on a set of consistent conventional numerical standards

– In addition local and regional gravity observations around the IVRS stations are required.

Regional and national height reference systems can be integrated into an IVRS by GNSS/levelling aligned to ITRF and using the CGGM and the numerical standards.

Changes of the solid and fluid Earth surface can be observed with respect to the conventional IVRF level by relevant observation techniques. The IVRS level is defined by a conventional W_0. The conventional IVRS level has to be related to the instantaneous mean sea surface level (MSSL).

Deficiencies

In view to a planned ISO registry for geodetic parameters, the establishment of an information system describing the various regional vertical reference frames and their relation to an IVRS was not realized. This includes the determination of transformation parameters between regional vertical reference frames and the unified global height system.

Further open topics are the relationships between an IVRS and the International Terrestrial Reference System (ITRS) (Basic relations between ITRS and IVRS conventions, parameters, realization, models).

Proposed continuation

The realization of an IVRS is a typical item of the IAG project GGOS, mainly as a combination of different products of IAG services.

The IAG has to clarfy inconsistencies in the numerical parameters for integrated geodetic applications. Conventions for the definition and realization of the parameters of the MSSL have also to be agreed.

Proposed items for continuation:
- Discussion of the results of ICP1.2 (GGOS action)
- Initiation of a pilot project for an IVRS realization on the basis of the IGS TIGA-PP, GGP and IGFS for AG and a CGGM (call for participation as an IGFS action)
- Further development of the CVRS conventions
- Decision about numerical standards as task of GGOS in cooperation with International Astronomical Union (IAU) and international hydrological associations.

The project continuation shall be realized, in cooperation with other organizations, especially the International Association of Hydrological Sciences (IAHS), the International Association for the Physical Sciences of the Oceans (IAPSO), the International Hydrographic Organisation (IHO), the International Federation of Surveyors (FIG), and. the Interservice Geospatial Working Group (IGeoWG) of NATO.

Johannes Ihde, Chair ICP1.2
June 15, 2007

Annex:

Numerical Standards

The Geodetic Reference System 1980 (GRS 80, 1980) defines major parameters for geodetic reference systems related to a level ellipsoid. It is agreed by the International Union of Geodesy and Geophysics (IUGG), International Association of Geodesy (IAG) and International Astronomical Union (IAU). The GRS80 parameters are recommended by IAG for the conversion of ITRF Cartesian coordinates to ellipsoidal coordinates. It is used worldwide for many map projections and million of coordinates are related to it.

At the IUGG General Assembly 1991 in Vienna new values for the geocentric gravitational constant GM and the semi-major axis a of the level ellipsoid were recommended. Since this time these parameters have been used in global gravity models e.g. EGM96. The two other defining parameters were not changed.

In the IERS 2003 conventions (Mc Carthy and Petit, 2004) numerical standards are listed (Table 1.1). These conventions have thew effect of standards and when read with chapters 4.1.4 and 4.2.5 recommended the use of GRS80 for transformations. The value of the geocentric gravitational constant (GM) has not changed since 1991. The parameters in Table 1.1 have the status of standards. In parallel in chapters 4.1.4 and 4.2.5 the GRS80 is recommended for transformations.

Table 1 contains parameters of different level ellipsoids. The gravitational constants GM of GRS80 and IERS 2003 conventions differ in the metric system by about 0.9 m. The semi-major axis of both standards differs by 0.4 m. It has to be stated, that the IERS 2003 conventions recommends different level ellipsoid parameters for different applications.

GRS80 is recommended (and generally used) for geometrical applications. For global gravity models, various inconsistent values are used in practice.

The IAG needs to remove this inconsistency to enable the development of integrated geodetic applications (cf., Hipkin, 2002). The geoid potential parameter W_0 of a Global Vertical Reference System defines the relationship of the physical heights to the Earth body. The parameter W_0 must be consistent between systems to ensure the relations to be reproducible.

Table 1. Level ellipsoid parameters

Ellipsoid	Semi-major axis a in m	Flattening f^{-1}	Geocentric gravitational constant GM in $10^8 m^3 s^{-2}$	U_0/W_0 in $m^2 \cdot s^{-2}$	γ_e in $m \cdot s^{-2}$
Int. Ell. 1930 (Hayford)	6 378 388	297	3 986 329		
GRS 67	6 378 160	298.247	3 986 030		
GRS 80	6 378 137	298.2572221 ± 01	3 986 005	6 263 6860.850	9.78032677
IUGG 91	6378136.3 ± 0.5		3 986 004.41 ± 0.01		
IERS 2003 Conventions (zero tide)	6378136.6 ± 0.1	298.25642 ± 0.00001	3986004.418 ± 0.008	62636856.0 ± 0.5	(9.78032666)*

Angular velocity of the Earth rotation ω in 10^{-11} rad s^{-1} 7 292 115

In addition to the existing IERS numerical standards other parameters shall be calculated and included in the IERS conventions e.g.
γ_e normal gravity at equator
γ_p normal gravity at pole
*   not consistent with IERS 2003 Conventions
(Remark: The numerical value of W_0 has to be revised in view of recent work done at the DGFI)

IAG Inter-Commission Project
ICP1.2 Vertical Reference Frames

Conventions for the Definition and Realization of a
Conventional Vertical Reference System (CVRS)

Draft 3.0

File name	CVRS_conventions_[Version]_[datum]			
Version	Chapter	Author	Date	Comments
1.0	all	JI	2006-04-07	
1.1	all	JI	2006-05-05	Review after Prague WS
1.2	all	JI	2006-07-10	
2.0	Foreword/Introduction/Scope	JI	2006_Sept.	
2.1	all	JI/StSh	2007-04-20	
3.0 final	all	JI	2007-06-29	Reviewed by: Matt Amos, Bernhard Heck, Bill Kersley, Tilo Schöne, Laura Sanches, Hermann Drewes

Preamble

At present, there are some hundred physical height systems realized worldwide. The realization of a global reference surface for physical height systems, the relation of individual tide gauge records with respect to the reference surface, the separation of sea level changes and vertical crustal movements at tide gauges, and the connection with the terrestrial reference system are unsolved problems. To proceed towards a unified physical height system we need at the centimetre accuracy level:

– a unique global height datum,
– consistent parameters, models and processing procedures for the Terrestrial Reference Frame (TRF) and gravity field,
– a closed theory for the combination of parameters (space techniques, gravity),
– consideration of time dependency,
– a rigorous concept for the realization.

Conventions for a global vertical reference system (GVRS) shall be a step forward to solve these problems.

Contents

Foreword

ICP1.2 is an Inter-Commission Project of Commission 1 *Reference Frames* and Commission 2 *Gravity Field* of the International Association of Geodesy (IAG).

The objectives of ICP1.2 are
- To elaborate a proposal for the definition and realization of a global vertical reference system (World Height System – WHS);
- To derive transformation parameters between regional vertical reference frames;
- To establish an information system describing the various regional vertical reference frames and their relation to a World Height Frame (WHF).

The program of activities includes a
- Harmonization of globally used height data sets;
- Study of combination procedures of height data sets from different techniques;
- Study of information on regional vertical systems and their relations to a Global Vertical Reference System for practical applications;
- Unification of regional (continental) height systems.

It was formed at the 2003 IAG General Assembly in Sapporo and is required to report back to the 2007 General Assembly in Perugia.

Introduction

The Earth's surface may be described by its geometry and the potential of the Earth gravity field. The determination of heights includes both of these aspects, the geometric part and the geopotential part. Presently, space geodetic techniques allow an accuracy in geometric positioning of about 10^{-9} of the Earth's radius in global and continental scales. Gravity field parameters, including the physical height components, can at present be determined only 2 to 3 orders of magnitude less accurately than the geometric parameters. Moreover, the current height reference frames around the world differ in their vertical datum (e.g., the mean sea-level at the fundamental tide gauges) and in the theoretical foundations of the height systems. There is no unified global (geopotential) height reference system defined and realized like the International Terrestrial Reference System (ITRS).

A considerable progress in the definition and realization of a global vertical reference system will be attained from the data of the new satellite altimetry and gravity field missions. Based on the classical and modern observations, the Project on Vertical Reference Frames shall study the consistent modeling of both geometric and gravimetric parameters, and provide the fundamentals for the establishment of a unified global vertical reference frame.

This document provides conventions for the definition and realization of a global Vertical Reference System (GVRS). The present draft is the result of the IAG Inter-Commission Project 1.2 Vertical Reference Frames. The VRS conventions are based on the IERS conventions 2003 (McCarthy and Petit, 2003) and are in agreement with resolutions of international scientific organisations.

Scope

The VRS conventions define the concept for the definition and realization of a global vertical reference system, which is consistent with the IERS conventions 2003. The VRS conventions are also valid for regional vertical reference systems and their relationships to a global VRS.

The conventions shall be used by the scientific geodetic community, national mapping agencies and companies for stationary, kinematic and dynamic tasks.

Normative references

The conventions for the definition and realization of a WHS shall be aligned to the IERS 2003 Conventions.

The IERS 2003 Conventions basically contain the current parameters and procedures for reduction of coordinates. Parameters and procedures for the requirements of a WHS shall be studied in detail within the next years.

The following documents are indispensable for the application of this document. For dated references, only the cited edition applies. For undated references, the latest edition of the referenced document (including any amendments) applies.
- ISO 1000, *SI units and recommendations for use of their multiples and of certain other units*
- ISO 19100 *Geographic Information standard family as far as useful.*

1.   General Definitions and Numerical Standards
1.1   Permanent Tide

A Vertical Reference System (VRS) is composed by geometric and gravity potential parameters, including their variations with time, in particular those generated by Earth tides. The different notions related to the treatment of the permanent tide are shown pictorially in Figures 1.1 and 1.2.
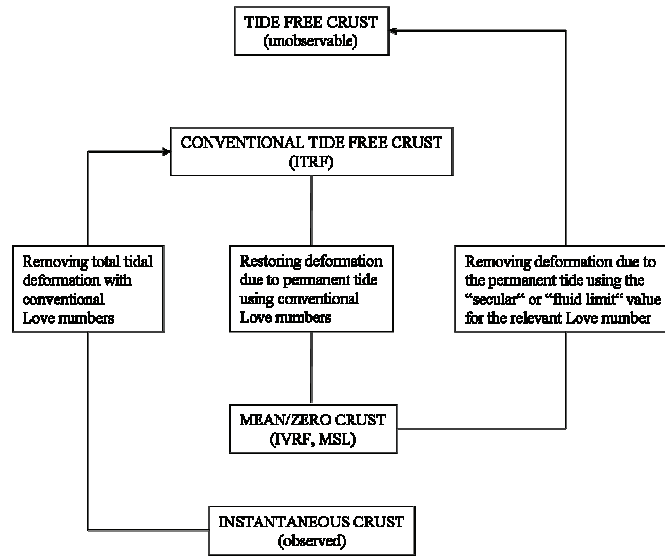

Figure 1.1   Treatment of observations to account for tidal deformations in terrestrial reference systems **(IERS conventions 2003)**
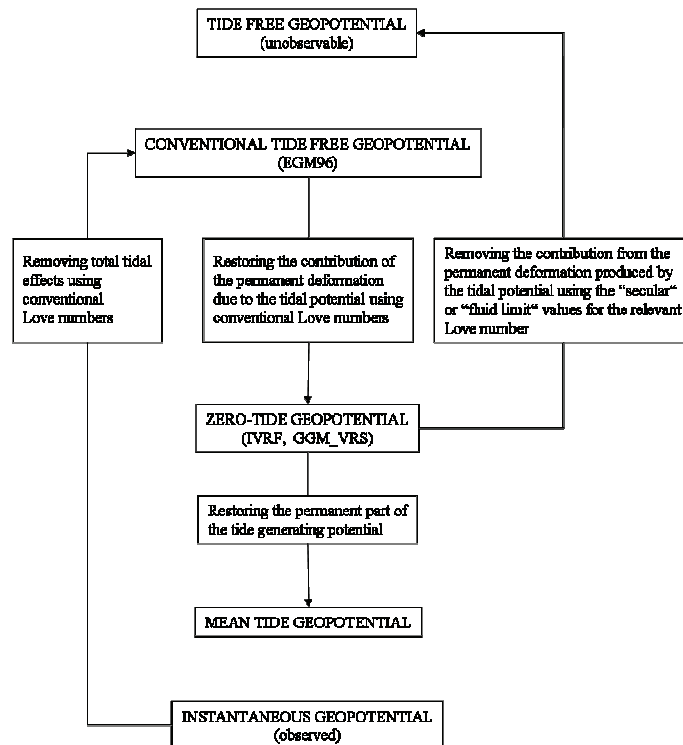

Figure 1.2   Treatment of observations for tidal effects in the geopotential **(IERS conventions 2003)**

The foundations of the IAG Resolution No. 16 adopted at the General Assembly in Hamburg in 1983 have not changed. The most adequate tide system applicable to both gravity acceleration and gravity potential of the rotating and deforming Earth is the zero tide system. The pendent for the geometry is the mean/zero tide crust concept. The mean sea surface corresponds to a crust deformed by mean/zero tides.

There is no justification for the application of a tide free concept for both the geometry and the gravity field. The tide free crust is far away from the real Earth shape and is unobservable. Even if the tide free concept is kept for the terrestrial reference system parameters, the IAG Resolution No. 16 adopted in Hamburg in 1983 shall be used for the gravity and geopotential.

The mean tide geopotential is not an alternative to the zero tide geopotential, because the condition of the Laplace equation is not fulfilled.

The IERS 2003 Conventions consider in Chapter 1.2 the treatment of the tidal effects in terrestrial reference systems and in the geopotential. For the vertical reference system Figures 1.1 and 1.2 have to be supplemented by the tidal conventions of the VRS.

1.2   Numerical Standards

The IERS numerical standards shall be valid for the VRS. The numerical standards listed in table 1.1 of the IERS 2003 conventions include all defining parameters for a level ellipsoid/mean Earth ellipsoid.

The level ellipsoid is the common reference level for spatial geodetic coordinates and Earth gravity field (geoid undulations and gravity anomalies). Therefore the level ellipsoid is of central importance for the combined analysis of space geodesy and gravity field. The basic relationships, parameters and formulae e.g. for normal gravity must be consistent and conventional for vertical and gravity reference systems.

2.   Conventional Vertical Reference System and Frame (CVRS, CVRF)

2.1   Concepts and Terminology

2.1.1 Basic Concepts

Height determination in general and therefore the realization of a Vertical Reference System relies on a natural combination of space geodesy and the Earth gravity field determination. A Vertical Reference System (VRS) is given by a geometrical and a physical component. The geometrical component is based on ellipsoidal heights and the corresponding reference surface: the ellipsoid. Since ellipsoidal heights can be only derived from geocentric coordinates X, the definition of the VRS geometrical component corresponds to the definition of a Terrestrial Reference System (TRS), complemented by the definition of a level ellipsoid. The VRS physical component is given in terms of geopotential quantities: geopotential values (W_p) or geopotential numbers (C_p) associated to a global unified reference level W_0. The defining potential gravity field, in the same manner as the TRS, co-rotates with the Earth in its diurnal motion in space. The realization of a such VRS is called Vertical Reference Frame (VRF) and it corresponds to a set of physical points (passive monuments or continuously operating stations) with precise geopotential values (W_p, or $c_p=W_0-W_p$) and geometrical coordinates X. The VRF includes the variation of the coordinates (X, W_p or c_p) with time (V_X, V_{Wp}, V_{Cp}).

Relationship between gravity field, geopotential field and the (geometrical) terrestrial reference system. Between the geopotential scalar field $W(X)$ and the outer Earth gravity vector field $\vec{g}(X)$ the following relationship is valid:

$$\vec{g} = grad\, W = -g \begin{pmatrix} \cos\Phi & \cos\Lambda \\ \cos\Phi & \sin\Lambda \\ & \sin\Phi \end{pmatrix} \quad , \quad g_P = g(X) = |\,grad\, W_P\,| = \left(-\frac{\partial W}{\partial H}\right)_P \quad (2.1.1\text{-}1)$$

with the natural coordinates astronomical latitude Φ, astronomical longitude Λ, and the potential of Earth gravity field W.

In a very general notation, the relationship (2.1.1-1) can be expressed as:

- $P(X, W, g) = P(X, W, -\partial W/\partial H)$ or

- $W(X) = W_P$ collocated with $g(X) = g_P = -\partial W_P/\partial H$.   (2.1.1-2)

The two fields are functions of time in a Euclidean space. Therefore the time dependence has to be considered:

- $W_p(t) = W_p^0 + \dot{W}_p^0(t - t^0)$,

- $g_P(t) = g_P^0 + \dot{g}_P^0(t - t^0)$ and   (2.1.1-3)

- $X_P(t) = X_P^0 + \dot{X}_P^0(t - t^0)$.

Ideal Vertical Reference Systems. An ideal Vertical Reference System is defined by the outer Earth's gravity potential field, W_p values of which can be represented as a function of a global gravity model and the geometrical coordinates X of each point P referred to an ideal Terrestrial

Reference System co-rotating with the Earth. For the Terrestrial Reference System the IERS Conventions 2003 are fully valid.

For practical use the geopotential values W_P at a point $P(X)$ may be related to a physical reference level represented by an equipotential surface of the Earth gravity field with a W_0 value. The geopotential numbers $c_P = W_0 - W_P$ are related to a conventional zero level W_0. A time dependency has to be considered $W^0(t) = W^0 + \dot{W}^0(t - t^0)$. The coordinates of the TRS have to be related to the equivalent geometrical reference surface defined by the parameters of the level ellipsoid. The level ellipsoid shall be conventional.

The vertical datum is an equipotential surface with a conventional value W_0 of the Earth gravity field potential. The vertical components are the differences ΔW_P between the potential W_P of the Earth gravity field at the considered point P and the potential of the VRS zero level W_0. The potential difference $-\Delta W_P$ is also designated as geopotential number $c_P = -\Delta W_P = W_0 - W_P$.

Conventional Vertical Reference System (CVRS). A CVRS is defined by the set of all conventions, algorithms and constants which provide the origin and scale of that system and their evolution with time.

Conventional Vertical Reference Frame (CVRF). A CVRF is the realization of a CVRS by a set of physical points (passive monuments or continuously operating sstations) with precisely determined geopotential numbers and geocentric coordinates X referred to the Conventional Terrestrial Reference System (CTRS)..

A CVRF is given by the set of all conventions, algorithms and constants which provide the geopotential codes and their evolution with time derived from geodetic observations. Two types of frames are currently distinguished, namely dynamical and kinematical frames, depending on whether or not a dynamical model is applied in the process of determining these coordinates.

2.1.2 International Vertical Reference System (IVRS)

The geometrical component of the International Vertical Reference System (IVRS) corresponds to the International Terrestrial Reference System (ITRS), complemented by the definition of a conventional level ellipsoid. The definition of the IVRS physical component fulfils the following conditions:

1. The vertical datum is defined as the equipotential surface of the Earth gravity field potential:

   $$W_0 = const. \tag{2.1.2-1}$$

   The vertical datum defines the relationship of the physical heights to the Earth body. W_0 shall be conventional and reproducible.

2. The unit of length is the meter (SI). The unit of time is the second (SI). This scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modelling.

3. The vertical coordinates are the differences $-\Delta W_P$ between the potential W_P of the Earth gravity field at the considered points P and the potential of the CVRS conventional zero level W_0. The potential difference $-\Delta W_P$ is also designated as geopotential number c_P:

$$-\Delta W_P = c_P = W_0 - W_P \,.[1] \tag{2.1.2-2}$$

The potential difference can directly be derived by levelling in combination with gravity reductions, or indirectly by applying the disturbance potential in connection with geodetic space observations

$$W_P = U_P + T_P \qquad \text{with} \qquad U_P = U_0 + \partial U_0/\partial h \cdot h + \ldots \,. \tag{2.1.2-3}$$

4. The CVRS is a zero tide system, in agreement with the IAG Resolution No 16 adopted at the General Assembly in Hamburg in 1983.

Remark: The definition W_0 = const. is fixed to a conventional value and therefore, it is time independent. However, this W_0 value has to be related to the Earth body.

The definition of W_0 is connected with the determination of the mean geoid following the Gauss/Listing definition.

2.1.3 The Realization of the IVRS – The International Vertical Reference Frame (IVRF)

A Vertical Reference Frame (VRF) as a realization of a Vertical Reference System (VRS) can be established by a combination of gravity field and geodetic space observations. The realization of all VRSs are of the type crust-based. One datum parameter is needed to fix a VRF to a given epoch: another parameter is needed to consider the time evolution of the vertical datum.

VRF should be determined either by analysis centres or by combination centres, and ultimately as products (see sub-section 2.2).

In general the determination of the potential of the Earth gravity field W_P at a point $P(X)$ on the Earth surface is feasible in two ways:

(1) By the determination of the disturbing potential T_P, as solution of the geodetic boundary value problem (BVP) at the known position $P(X)$ added to the normal potential U_P at the same point:

$$W_P = U_P + T_P \,. \tag{2.1.3-1}$$

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[1] For practical use the geopotential numbers are transformed into different types of physical heights: for instance, the normal height is derived by $H_P^N = c_P/\bar{\gamma}$, where $\bar{\gamma}$ is the mean normal gravity between the level ellipsoid and the related point P$_Q$. The orthometric height corresponds to $H_P^0 = c_P/\bar{g}$ where $\bar{g}$ is the mean gravity between geoid and Earth surface.

With $U_P = U_0 + \partial U_0/\partial h \cdot h = U_0 - \bar{\gamma} \cdot h$ follows $W_P = U_0 - \bar{\gamma} \cdot h + T_P$ and further more $\bar{\gamma} \cdot h = (U_0 - W_0) + (W_0 - W_P) + T_P = (U_0 - W_0) + c_P + T_P$. Then, the following relationship is obtained:

$$c_P = - (U_0 - W_0) + \bar{\gamma} \cdot h_P - T_P.$$ (2.1.3-2)
$$H^N = - (U_0 - W_0)/\bar{\gamma} + h_P - T_P/\bar{\gamma}.$$

On the geoid *G,* the special case $W_P = W_G = W_0 = U_G + T_G = const.$ is given. This is the classical approach of Physical Geodesy.

With $U_G = U_0 + \partial U_0/\partial h \cdot N = U_0 - \gamma_0 \cdot N$ and $W_0 = U_0 - \gamma_0 \cdot N + T_G = const.$ it can be written:

$$N = (U_0 - W_0)/\gamma_0 + T_G/\gamma_0.$$ (2.1.3-3)

(2) By the geopotential numbers c_P derived from geometrical (spirit) levelling in combination with gravity reductions. In this case, if the potential of the height reference surface of a VRF *k* is W_{0k}, the gravity potential W_p at P reads:

$$W_P = W_{0k} - c_{Pk}.$$ (2.1.3-4)

With $W_P = W_{0k} - c_{Pk} = W_0 - c_P$ we get the geopotential number in relation to W_0

$$c_P = c_{Pk} + W_0 - W_{0k}$$ (2.1.3-5)

Spirit levelling is the classical approach of realizing a VRF. The second way can be used on continents, it is a regional approach. Generally, a globally homogenous VRF can be established by the first procedure.

The combination of both ways (determination of the anomalous potential and spirit levelling with gravity reductions, see Annex A1) is useful for the determination of the level of the regional height reference surface W_{0k} with respect to the global level W_0 :

$$W_{0k} - c_{Pk} = U_P + T_P = U_0 - \bar{\gamma} \cdot h + T_P$$ (2.1.3-6)

$$W_{0k} = U_0 - \bar{\gamma} \cdot h + c_{Pk} + T_P.$$ (2.1.3-7)

The disturbing potential T_P at any point *P(X)* on the Earth's surface can be determined by solving the geodetic BVP and integrating gravity over the whole Earth's surface σ:

$$T_P = \frac{R}{4\pi} \iint_\sigma (\Delta g + G_1 \cdots) S(\psi) d\sigma$$ (2.1.3-8)

or by applying a global gravity model (GGM) to obtain the real gravity potential W_p at the point P(X).. From Eq. (2.1.3-1), T_p corresponds to:

$$T_P = W_P - U_P$$

being W_P, in terms of a spherical harmonic series:

$$W = \frac{GM}{r}\left[1 + \sum_{n=1}^{n_{max}}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right]P_{nm(\cos\theta)}\right]$$
$$+ \frac{1}{2}\omega^2 r^2 \cos^2(90° - \theta).$$

$$(2.1.3-9)$$

In this case, it is necessary to consider high frequency components of the regional Earth gravity field T_{Preg} for higher precision, i.e.:

$$T_P = T_{GGM} + T_{Preg}.$$

$$(2.1.3-10)$$

The geopotential number corresponds to the integration of the level differences obtained from spirit levelling by gravity over the height from the vertical reference surface of a VRF k to the point $P(X)$ on the Earth's surface, along the levelling way

$$c_{p_i} = W_{0i} - W_p = -\int_{0i}^{p} dW = \int_{0i}^{p} gdh .^2$$

$$(2.1.3-11)$$

A classical task for reducing observations from the Earth surface to the reference ellipsoid was the combination of levelling and geoid determination: $\bar{\gamma} \cdot h_P = (U_0 - W_0) + c_P + T_P$, where $U_0 = W_0$ was assumed. In space geodesy this approach is not necessary.

The combination of different geometric and gravimetric observables is only possible; if the data of the gravity field and of space geodesy are related to a consistent set of parameters. (see also Annex A2).

The instantaneous mean sea surface should be observed agains a conventional W_0 value, which has been agreed as the zero level of a GVRS. A conventional W_0 value has to be consistent with other defining parameters of Earth models. IAG shall agree upon a unique sets of parameters.

The determination of the instantaneous mean sea surface (MSS) level needs further specifications (see also Annex A3). The International Hydrographic Organization (IHO) defines the mean sea surface level MSL as the average height of the sea surface at a tide station for all stages of the tides over a 19 year period usually determined from hourly height readings measured with respect to a fixed pre-determined reference level (Chart Datum). For the global MSS, it has to be defined over which area the individual MSLs shall be averaged. A natural limit are the ice-free zones of the oceans. Because of the changes of the MSS with time, an epoch for the average level has to be defined.

For the approximation of the global MSS, models with suitable resolution and accuracy shall be used. The individual MSL measurements shall be averaged at least using bins half to the resolution of the GGM

AS a consequence,the following conventions for a global MSS level model are proposed:
-   The MSS is the average of the sea surface heights of the global ocean.

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[2] The height anomaly can be derived with Bruns theorem from the disturbing potential T_P by $\zeta = T_P / \gamma_Q = (W_P - U_P)/\gamma_Q$, where γ_Q is the normal gravity at the telluroid point PQ. The combination of the geometric heights h_P from 3D positioning with normal heights H_P^N and height anomalies ζ_P follows the simple relationship $H_P^N = h_P - \zeta$. A compatible relationship exists between geometric heights, orthometric heights and geoidal heights $H_P^O = h_P - N$.

- It shall be referred to the epoch 2000.0.[TS1]
- It shall be reduced to the level of the zero tide system.

Currently the global MSS cannot be derived with the globally unique accuracy needed, therefore, for the time being the MSS is limited to ±68° latitude

2.2   IVRF products

It is the goal to use the available infrastructure of the IAG services, data bases, and standards as far as possible. A GVRS should be realized in context with a global integrated network, which combines at terrestrial reference stations the geodetic space techniques, highly precise absolute and relative gravity, levelling with gravity reductions, and tide gauges with permanent or episodic observations.

IVRS Network

Integrated networks, which combine at terrestrial reference stations geodetic space techniques, highly precise absolute and relative gravity, levelling with gravity reductions, and tide gauges with permanent or episodic observations and supplementary information (meteorological parameters, surrounding information of the stations, e.g. eccentricities and ground water level) are recently under discussion, and partly in realization. They are the basis to combine the geometric and height reference systems with Earth gravity field parameter estimation. These activities are in agreement with the IAG Global Geodetic Observing System (GGOS, Rummel et al., 2002). In Europe the IAG project of the realization of a European Combined Geodetic Network (ECGN) is in progress (Ihde et al., 2005.1, 2005.2). In South America, a similar network is being faced by SIRGAS (Sánchez and Brunini 2006).

The objectives of this kind of networks are:

- maintenance of long time stability of the terrestrial reference system with an accuracy 10^{-9} especially in the height component
- in-situ combination of geometric positioning (GNSS) with physical heights and other Earth gravity parameters at 1 cm-accuracy level
- modelling of influences of time dependent parameters of the solid Earth, the atmosphere, the oceans, the hydrosphere, and the Earth gravity field, for different applications of positioning
- contribution to global gravity field modelling.

The combination of various geodetic methods contributes especially to the vertical component and the determination of vertical velocities v. In a first approximation, it can be assumed that the disturbing potential is time independent, it means

$$v_h = v_H \tag{2.2-1}$$

The velocities of the physical heights H can be derived from time series of the ITRFxx heights h:

$$H_P(t) = H_P^0 + \dot{h}_P^0(t - t^0) . \tag{2.2-2}$$

An integrated GVRF network which combines the stations of the IGS TIGA Project (Tide Gauge Benchmark Monitoring Project) with the network of superconducting gravimeters of the IAG Global Geodynamic Project (GGP) could be a beginning of a GVRF. (Fig. 2.1)
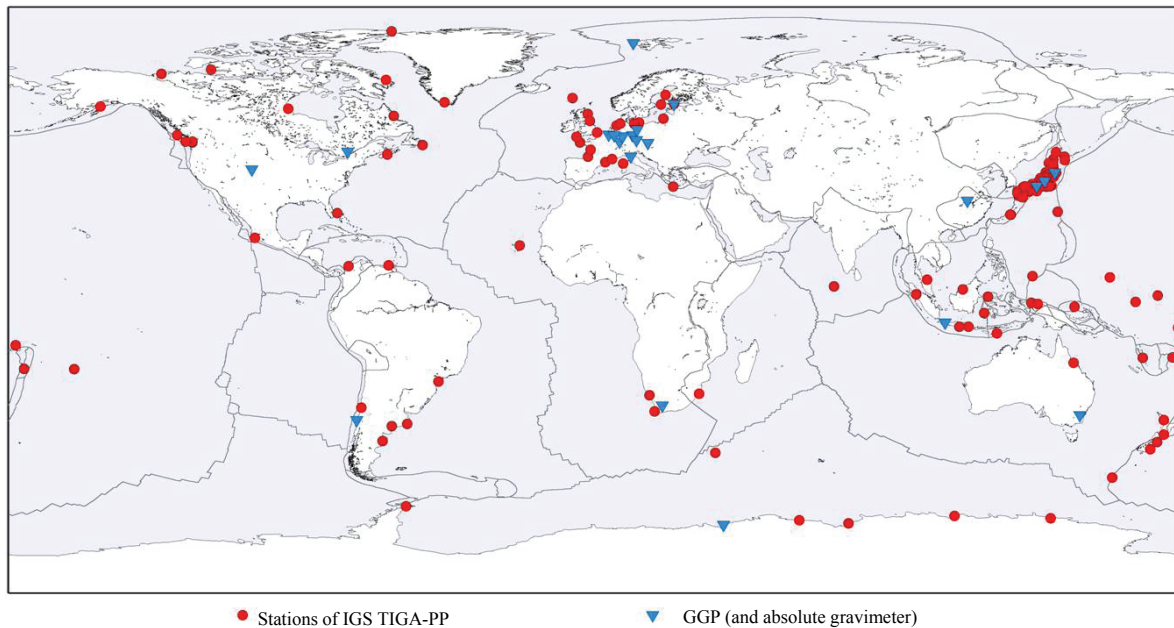
● Stations of IGS TIGA-PP   ▼ GGP (and absolute gravimeter)

Fig. 2.1 GVRF, an integrated network

IVRF products should mainly be coordinated combinations of existing IAG services and products. The following types of stations should be part of the IVRF network:

- ITRF core stations with co-location of geodetic techniques (ITRFCS)[3]

- GGP stations co-located with absolute gravimeter measurements and IGS stations

- PSMSL stations, co-located with IGS stations (i.e. TIGA sites)

- Reference tide gauges for chart datums

- Reference stations of levelling networks co-located with IGS stations

The required parameters for each station are: coordinates and velocities in the current ITRFxx one (or both) geopotential coordinate T_P, or c_P and, at tide gauges, the mean sea surface or mean sea surface topography and their changes. Under the condition $\dot{T} = 0.$, the velocities of the physical heights can derived from the ITRFxx heights: $\dot{H} = \dot{h}$.

Global gravity model GGM

(Product of the International Centre for Global Earth Models - ICGEM in the framework of the International Gravity Field Service - IGFS)

The used GGM should be a satellite only solution to exclude inconsistencies from local gravity data. It should internationally be agreed and conventional (CGGM). To reach a one centimetre accuracy level, the GGM has to be augmented with local or regional gravity data.

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[3] Co-location of a minimum of two of the three space techniques: SLR, VLBI, GNSS and Doris.

Model of the Sea Surface or Sea Surface Topography

(Product of the International Altimeter Service - IAS))

To use tide gauge observations for VRF unification information about the absolute sea surface topography (SSTop) around the tide gauges is necessary. Since the SSTop at coasts is influenced by local effects and satellite altimeter measuremnents on these zones can not at present be used with high precision, the use of offshore tide gauges should be considered. Of course, a precise geoid on the basis of a CGGM is necessary to connect the offshore tide gauges with the VRF. For further consideration: The integration of worldwide GNSS Tsunami early warning system buoys for long term control of mean sea surface and connection to satellite altimeter observations can provide an additional progress.

3. Unification of VRF - Relationships between IVRS, Regional, and Local VRS

3.1   Continental and Regional VRF

There are about a hundred different continental and regional vertical reference systems worldwide, related to different tide gauges or bench marks, realized by spirit levelling as static systems, and reduced for the gravity effect by different models. The tide gauges are related to local mean sea level (MSL) determined at an arbitrarily selected epoch which differs from a mean global equipotential surface (the geoid) up to the order of two meters.

The accuracy of the heights in these systems is limited regionally by the error propagation of spirit levelling and globally by the datum realization with different tide gauges and different epochs by 10^{-6} to 10^{-7}. The repetition rate of physical height determination is in general only 10 to 50 years.

An example of a continental datum is in Europe the Normaal Amsterdams Peil (NAP). The NAP level was derived from tide gauge observations in 1684 and is related to the mean height tide level. The level was transferred over hundreds of years by bench marks. The NAP level is not in agreement with the present sea level.

The accuracy of present realizations of physical height reference systems in a global scale is about two orders of magnitude less than that of the ITRF. The repetition time rate differs by one order of magnitude. In the frame of global geodesy at centimetre-level, physical heights appear as inconsistent elements.

The use of the terrestrial reference frame, e.g. the ITRF, for physical height determination requires the transformation of the geometric heights, related to the ellipsoid, to physical heights referring to its reference surface (geoid or quasigeoid, respectively). To keep the ITRF position accuracy, one has to know the height reference surface with the same accuracy, which is not feasible at present.

3.2   Chart datums

Hydrographers and nautical cartographers have been surveying and producing nautical charts for over 200 years. These charts show the depth of water that exists in any given place. Clearly these depths need to be referenced to some vertical datum level. As the majority of charts produced are intended for marine navigational use, nautical cartographers have, in the interests of safety of navigation, chosen a datum level such that the depth of water indicated at any given place will be the least depth of water that should be encountered there. This datum level is referred to as "Chart Datum" and the International Hydrographic Organization (IHO) has resolved that this should be the Lowest Astronomical Tide (LAT) which they define as:

LAT is defined as the lowest tide level which can be predicted to occur under average meteorological conditions and under any combination of astronomical conditions. It is recommended that LAT be calculated either over a minimum period of 19 years using harmonic constants derived from a minimum of one year's observations or by other proven methods known to give reliable results.

Thus the value of Chart Datum is determined by analysing tidal observations, in a given place, over a long period of time and is set such that it lies just below the lowest low tide that can be predicted for that place[4]. The value of Chart Datum, when determined, is connected to the land

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[4] In areas where there is a minimal range of tide it is customary to use Mean Sea Level (MSL) as the Chart Datum

levelling system. One of the issues with Chart Datum is that it varies both spatially and temporally. The collection of hydrographic data is a slow and expensive process and thus much of the data held is quite old. The advent of digital computers has led to the requirement to merge both marine and terrestrial data sets into a unified project. Given the spatial and temporal variations in Chart datum this is a not a trivial matter. Consequently hydrographers and nautical cartographers seek the establishment of a Global Vertical Reference Frame through which vertically referenced marine and terrestrial data can be merged whilst still allowing data to be related to Chart Datum for safety of navigation uses. Further information is available in the Handbook "Developing a vertical reference surface for hydrography" prepared by a Working Group of FIG Commission 4.

3.3   General Relationships

For the unification of Earth gravity related height reference systems generally and globally applicable is the unification of a global gravity model (GGM) and GNSS/levelling points with ellipsoidal heights h and levelling heights reduced by gravity effects i.e. $H_P^N = c_P / \overline{\gamma}$. At the GNSS/levelling points, a high resolution gravity model related to a GGM is useful $T_P = T_{GGM} + T_{Preg}$. To realize a homogeneous vertical datum unification, a specific GGM should be recommended as conventional model. In general, there are two approaches possible:

i General case for realization and unification: combination of GNSS and GNSS/levelling with a GGM

The VRS realization at single points P is very simple and well known:

$$W_P = U_0 + \partial U_0 / \partial h \cdot h_{ITRF} + ... + T_{P\,GGM} + T_{Preg} \tag{3.3-1}$$

or

$$H_P = h_{ITRF} - \zeta_{P\,GGM} \quad . \tag{3.3-2}$$

In many cases, the hight anomalies ξ or the disturbing potential T are based on a GGM which is improved by local gravity and related to GNSS/levelling data. With such regional quasigeoid solutions a unique GVRS cannot be realized.

The transformation of geopotential numbers of regional VRF in a GVRF is possible if the level W_{0k} of a regional VRF k is known:

$$W_P = W_{0k} - c_{Pk} . \tag{3.3-3}$$

With a GGM and GNSS positions, the potential at levelling points can be determined in a global system

$$W_P = U_P + T_{P\,GGM} \text{ with } U_P = U_0 + \partial U_0 / \partial h \cdot h$$

$$W_P = U_0 + \partial U_0 / \partial h \cdot h_{ITRF} + T_{P\,GGM} . \tag{3.3-4}$$

The mean potential W_{0k} of the zero level of the regional VRF k and the difference to a global level W_0 can approximately be derived by:

$$W_{0k,i} = U_0 - \gamma_0 (h_{i,ITRF} - H_{k,i} - \zeta_{i\,GGM})$$
$$W_{0k} = mean\,W_{0k,i} \quad h \text{ and } \zeta \text{ shall be global} \tag{3.3-5}$$
$$\Delta W_{0k} = W_{0k} - W_0$$

By combination with geopotential numbers of the regional VRF, the level of a regional VRF k can be derived in single points i

$$W_{0k,i} = U_0 + \partial U_0 / \partial h_i \cdot h_{i\,ITRF} + T_{Pi\,GGM} + c_{Pk,i} \, .$$

The height of the zero level of the regional VRF can be derived for single points by

$$H_{0,VRF} = h_{i,ITRF} - H_{i,VRF} - \zeta_{i,GGM} \, . \tag{3.3-6}$$

To reach globally a one-centimetre accuracy level, the GGM has to be augmented with local or regional gravity data. To guarantee a homogenous unification the GGM shall be conventional: CGGM. h and ζ shall be global. (Fig. 3.1)
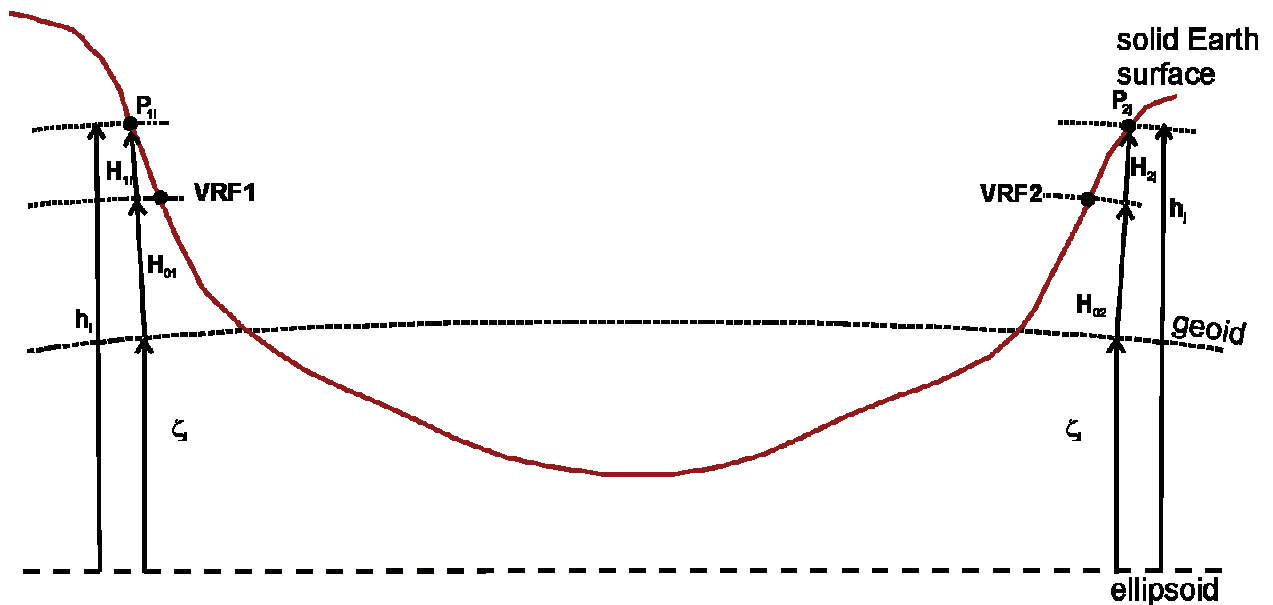

Fig. 3.1: Principle of VRF unification using GNSS/levelling and CGGM

ii. Unification by tide gauge observations

To use tide gauge observations for VRF unification, information about the absolute sea surface topography (SSTop) model and additional local modification of the SSTop around the tide gauges are necessary. At coasts, the SSTop is influenced by local effects and satellite altimeter measurements at those zones can not at present be used with high precision. Therefore the use of offshore tide gauges should be considered. Of course, a precise quasigeoid on the basis of a CGGM is necessary to connect the offshore tide gauges with the VRF. Similar to the relationship (3.3-6), the height of the zero level of the regional VRF can be derived for single points by (Fig. 3.2):

$$\begin{aligned} H_{0,VRF} &= \left(h_{MSS}^{TG} - \zeta^{TG} \right) - H_{MSS}^{TG} + \Delta H_{TG} \\ &= H_{MST}^{Mod} - H_{MSS}^{TG} + \Delta H_{TG} \, . \end{aligned} \tag{3.3-7}$$

$\left(h_{MSS}^{TG} - \zeta^{TG} \right) = H_{MST}^{Mod}$ shall be global.
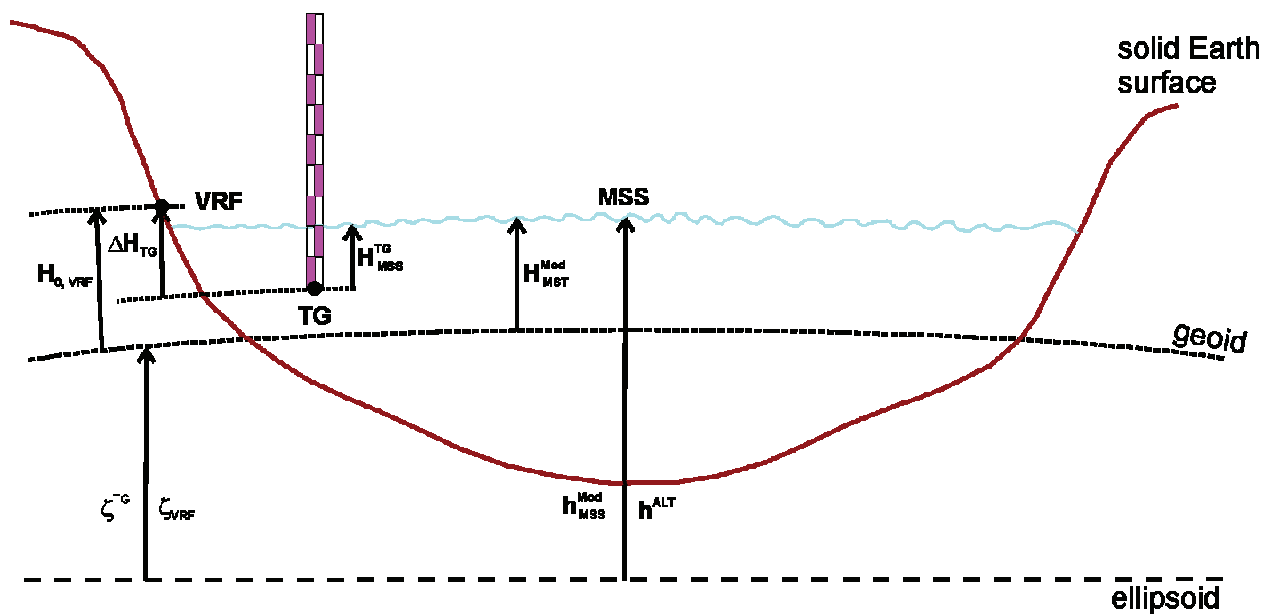
Fig. 3.2:   Principle of GRF unification using tide gauge observations and a global model of sea surface topography

References

Geodetic Reference System 1980 (GRS80) – In: Bulletin Géodésique, vol. 54, no. 3 (1980), 395-405.

Ihde, J., Baker, T., Bruyninx, C., Francis, O., Amalvict, M., Kenyeres, A., Mäkinen, J., Shipman, S., Ŝimek, J., Wilmes, H., 2005.1: Development of a European Combined Geodetic Network (ECGN). In: Journal of Geodynamics 40 (2005) 450-460.

Ihde, J., Baker, T., Bruyninx, C., Francis, O., Amalvict M., Luthardt, J., Liebsch, G., Kenyeres, A., Makinen, J., Shipman, S., Ŝimek, J., Wilmes, H., 2005.2: The Implementation of the ECGN Stations – Status of the 1st Call for Participation. In: EUREF Publication No. 14, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, Band 35, 49-58.

McCarthy, D. D. and Petit, G., 2004: IERS Conventions (2003). (IERS Technical Note; 32), Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2004.

Rummel, R., Drewes, H., Beutler, G., 2002: Integrated Global Geodetic Observing System (IGGOS): A candidate IAG project. Proceedings of the IAG 2001 Scientific Assembly, Budapest, Hungary, IAG Symposia, Vol. 125, 609-614, Springer, Berlin.

Sánchez, L, Brunini, C (2006). Current status and future activities of the subcommission 1.3b (South and Central America) of the IAG Commission 1 on Reference Frames. Presented at the IAG Symposium GRF2006 (Geodetic Reference Frames 2006). Munich, Germany. 2006-10-13. In press.

Annexes

A   Relationships

A1   GPS/Levelling

From **GNSS and levelling** observations on the solid Earth surface the disturbing potential can be derived:

$$T_P = - (U_0 - W_{0k}) + \bar{\gamma} \cdot h - c_{Pk} . \tag{A1-1}$$

If the difference $(U_0 - W_{0k})$ is not known then it is part of the GPS/levelling disturbing potential T_{Pk}:

$$T_{Pk} = T_P + (U_0 - W_{0k}) = + \bar{\gamma} \cdot h - c_{Pk} . \tag{A1-2}$$

The satellite **altimeter observations** h_S over the liquid Earth surface S represent a special case of GNSS/levelling. If $P = S$, the sea surface topography has to be considered:

$$T_S = T_G = W_0 - U_0 + \bar{\gamma} \cdot h_S + \Delta W_{SSTop} . \tag{A1-3}$$

From GNSS observations and the disturbance potential the **geopotential number** can be derived by (way 1, see above):

$$c_P = - (U_0 - W_0) + \bar{\gamma} h - T_P \tag{A1-4}$$

with altimeter observations h_S in connection with the disturbing potential T_S the **sea surface topography** can be derived:

$$- \Delta W_{SSTop} = W_0 - U_0 + \gamma_0 \cdot h_S - T_S . \tag{A1-5}$$

A2   Relationships between IVRS and the International Terrestrial Reference System (ITRS)

Basic relations between ITRS and IVRS/WHS (conventions, parameters, realization)

ITRS	WHS/IVRS
IUGG Resolution No. 2, Vienna 1991 IERS conventions (2003)	IAG ICP1.2 Vertical Reference Frames

origin

Explicit	Geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere.(At present no convention related to the motion of the geocentre)	Implicit

orientation

Initial BIH orientation. Non-rotating system. No global residual rotation with respect to horizontal motions at the Earth´s surface.	No necessary convention

units-scale

SI unit meter The ITRS scale consistent with the Geocentric Coordinate Time (TCG)	SI units meter and seconds The reference level $W_0 = const$ is realized by the combination of a conventional mean sea surface model with a conventional global gravity model following the geoid definition formulated by Gauss-Listing. This W_0 value divided by the normal gravity γ provides, in a very good approximation, the scale of the Earth body.

coordinates

quasi – Cartesian system X	potential of the Earth gravity field $W_P = W(X)$ $\quad = U_P + T_P \quad$ (GPM) $\quad = W_0 - c_P \quad$ (Levelling)

system parameters

mean Earth ellipsoid
(U_0, GM, J_2, ω)

realization

ITRFxx conventional tide-free	For example, for Europe: EVRF 2000 (UELN 95/98, ETRS89) $W_P = W_{NAP} + c_P$ (Levelling) Zero tidal system GRS 80

For South America:

Reference level:
$W_0 = 62\ 636\ 853{,}4 \pm 0{,}04\ \text{m}^2\ \text{s}^{-2}$

with standard deviation for P=1: ± 6,49 m^2 s^{-2}

Derived from:

GGM: EIGEN-GL04S (GRGS/GFZ),
                    n =150,
                    epoch 2000.0
                    zero tide system

MSS-model: From T/P (AVISO Data),
                    between φ = 60°N/S,
                    resolution 1° x 1°,
                    epoch 2000.0
                    zero tide system

Constants: GM = 398 600,4415 x 109 m^{3}s^{-2}

A3   Conditions for W_0 determination.

There are several options to align the zero Earth gravity field potential W_0 to the Earth surface/body by minimizing the disturbance potential over

- the geoid G of the whole Earth σ

$$\iint_\sigma T_G^2 d\sigma = \min, \quad \iint_\sigma T_G d\sigma = 0 , \qquad\qquad (W_0 = U_0) \qquad\qquad \text{(A3-1)}$$

  and equivalently

$$\iint_\sigma N^2 d\sigma = \min, \quad \iint_\sigma N d\sigma = 0 , \qquad\qquad\qquad \text{(H+M, p. 214, 5-74)}$$

- the sea surface S

$$\iint_S T_S^2 dS = \min , \iint_S T_S dS = 0 , \qquad\qquad\qquad\qquad \text{(A3-2)}$$

- the whole solid and fluid Earth surface σ

$$\iint_\sigma T_p^2 d\sigma = \min, \quad \iint_\sigma T_p d\sigma = 0 ,$$

  equivalently

$$\iint_\sigma (W_P - W_0)^2 d\sigma = \min , \iint_\sigma (W_P - W_0) d\sigma = 0 . \qquad\qquad \text{(A3-3)}$$

With the first solution we get the mean Earth ellipsoid as the best fitting ellipsoid to the geoid. This is the classical definition of the Gauß-Listing geoid and is the approach for the approximation of the mean Earth ellipsoid. The second solution gives the best fitting geopotential to the sea surface. The second and the third options have the sea surface as an identical part of the integration surface, and therefore an identical integrand.

For the second option an additional condition shall be valid:

$$\iint_S (h_{MSS} - N)^2 dS = \min , \iint_S (h_{MSS} - N) dS = 0 , \qquad\qquad \text{(A3-4)}$$

where the mean sea surface topography is

$$H_{SSTop} = h_{MSS} - N - (U_0 - W_0)/\bar{\gamma} .$$

The W_0 value can be defined and determined as mean value of the Earth gravity potential of the mean sea surface W_S over a defined area S of the open sea: a defined time period and related to an epoch:

$$W_{0S} = 1/S \iint_S W_S \ dS .$$

W_S of mean sea surface can be derived using a Global Gravity Model (GGM) expressed by a spherical function and sea surface heights h_S derived by satellite altimeter observations (ALT)

$$W_s = \frac{GM}{r_s}\left[1 + \sum_{n=1}^{n_{max}} \left(\frac{a}{r_s}\right)^n \sum_{m=0}^n [C_{nm}\cos m\lambda + S_{nm}\sin m\lambda] P_{nm(\cos\theta)} \right] = U_0 + \partial U_0/\partial h \cdot h_{s\,ALT} + T_{S\,GGM}$$
$$+ \frac{1}{2}\omega^2 r_s^2 \cos^2(90° - \theta)$$

If the sea level is long term changing it is useful to fix W_0 of the GVRS. W_{0S} can be observed against the conventional W_0 of the GVRS.

B   Related Standards

B1   Level ellipsoid of Somigliana/Pizzetti

Gravity Formula

Somigliana's closed formula for normal gravity is

$$\gamma_O = \frac{a\gamma_e \cos^2\phi + b\gamma_p \sin^2\phi}{\sqrt{a^2 \cos^2\phi + b^2 \sin^2\phi}} \ .$$

(B1-1)

For numerical computations, the form

$$\gamma_O = \gamma_e \frac{1 + k\sin^2\phi}{\sqrt{1 - e^2 \sin^2\phi}} \ ,$$

(B1-2)

with the values of γ_e, k, and e^2 shown above, is more convenient. ϕ denotes the geographical latitude.

The series expansion

$$\gamma_O = \gamma_e \left(1 + \sum_{n=1}^{\infty} a_{2n} \sin^{2n}\phi \right)$$

(B1-3)

with

$$a_2 = \frac{1}{2}e^2 + k \ , \qquad\qquad a_6 = \frac{5}{16}e^6 + \frac{3}{8}e^4 k \ ,$$

$$a_4 = \frac{3}{8}e^4 + \frac{1}{2}e^2 k \ , \qquad\qquad a_8 = \frac{35}{128}e^8 + \frac{5}{16}e^6 k$$

becomes

$$\begin{aligned}
\gamma_O = \gamma_e (1 &+ 0.005\,279\,0414\ \sin^2\phi \\
&+ 0.000\,023\,2718\ \sin^4\phi \\
&+ 0.000\,000\,1262\ \sin^6\phi \\
&+ 0.000\,000\,0007\ \sin^8\phi)\ ;
\end{aligned}$$

(B1-4)

it has a relative error of 10^{-10}, corresponding to $10^{-3}\ \mu ms^{-2} = 10^{-4}$ mgal.

The conventional series

$$\gamma_O = \gamma_e \left(1 + f * \sin^2\phi - \frac{1}{4}f_4 \sin^2 2\phi \right)$$

(B1-5)

with

$$f_4 = -\frac{1}{2}f^2 + \frac{5}{2}f m$$

(B1-6)

becomes

$$\gamma_O = 9.780327 \left(1 + 0.0053024 \sin^2\phi - 0.0000058 \sin^2 2\phi \right) ms^{-2} \ .$$

(B1-7)

C   Bibliography

Ardalan, Alireza A., 2005: A proposal for precise orthometric height computation

Balasubramania, N., 1994: Definition and realization of a global vertical datum. Department of Geodetic Science and Surveying, Ohio State University, Report No. 427, Columbus, Ohio.

Burša, M., Demianov, G. V., Yurkina, M. I., 1998: On the determination of the Earth's model – The mean equipotential surface. Studia Geophys. et Geod., 42, 467-471, Prague.

Burša, M., Groten, E., Kenyon, S., Kouba, J., Radéj, K., Vatrt, V., Vojtíšková, M., 2002: Earth dimension specified by geoidal geopotential. Studia Geophys. et Geod., 46, 1-8, Prague.

Burša, M., Kenyon, S., Kouba, J., Raděj, K., Vatrt, V., Vojtíšková, M., Ŝimek, J., 2002: World Height System specified by geopotential at tide gauge stations. Proceedings of the Symposium on Vertical Reference Systems, Cartagena, Colombia, IAG Symposia, Vol. 124, 291-296, Springer, Berlin.

Drewes, H., Sánchez, L., Blitzkow, D., Freitas, S., 2002: Scientific foundations of the SIRGAS vertical reference system. In: Drewes et al. Eds. IAG Symposia 124: Pp. 297-301, Springer.

Heck, B.: Problems in the definition of vertical reference frames. In: Sanso, F. (ed): V Hotine-Marussi Symposium on Mathematical Geodesy, Matera, Italy 2003. IAG Symposia Vol. 127 (2004), 164-173, Springer Berlin

Hernández, N., Blitzkow, D., Luz, Roberto T., Sánchez, L., Sandoval, P., Drewes H., 2001: Connection of the vertical control networks of Venezuela, Brazil and Colombia. In: Drewes et al. Eds. IAG Symposia 124: 324-327, Springer.

Hipkin, R., 2002: Is there a need for a Geodetic Datum 2000? Discussion of a "Heiskanen & Moritz" Proposition. Proceedings of the IAG 2001 Scientific Assembly, Budapest, Hungary, IAG Symposia, Vol. 125, 124-127, Springer Berlin.

Ihde, J., Augath, W., 2001: An vertical reference system for Europe. Veröffentlichung der Bayerischen Kommission für die Internationale Erdmessung, Nr. 61, 99-110, München.

Ihde, J., Augath, W., 2002: The European Vertical Reference System (EVRS), Its relation to a World Height System and to the ITRS. Proceedings of the IAG 2001 Scientific Assembly, Budapest, Hungary, IAG Symposia, Vol. 125, 78-83, Springer Berlin.

Ihde, J., Baker, T., Bruyninx, C., Francis, O., Amalvict, M., Kenyeres, A., Mäkinen, J., Shipman, S., 2004: Concept and Status of the ECGN Project. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Toledo, 4-7 June 2003, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, volume 33, pp. 57-65, Frankfurt am Main 2004.

Ihde, J., Baker, T., Bruyninx, C., Francis, O., Amalvict, M., Kenyeres, A., Mäkinen, J., Shipman, S., 2005: The implementation of the ECGN Station Status of the 1st Call for Participation. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Bratislava, 2-5 June 2004, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, volume 35, pp. 49-58, Frankfurt am Main 2005.

Ihde, J., 2005: Status of the ECGN project. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Vienna, 1-4 June 2005.

Ihde, J., Sánchez, L., 2005: A unified global height reference system as a basis for IGGOS. In: Journal of Geodynamics 40 (2005) 400-413.

Ihde, J., Sacher, M., Mäkinen, J., 2006: European Vertical Reference System (EVRS) 2007 – a Combination of UELN and ECGN. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Riga, 14-17 June 2006

Kaniuth, K., P. Hafäle, L. Sánchez, 2001: Subsidence of the permanent GPS station BOGA. In: Drewes et al. Eds. IAG Symposia 124: Pp. 56-59, Springer.

Mäkinen, J., 2004: Some remarks and proposals on the re-definition of the EVRS and EVRF. TWG of the IAG subcommission for the EUREF Bratislava.

Martínez, W., Sánchez, L., 2001: Improving the quasigeoid model in Colombia. In: Drewes et al. Eds. IAG Symposia 124: Pp. 152-155, Springer.

Rapp, R. H., Balasubramania N., 1992: A conceptual formulation of a World Height System. Report No. 421, Department of Geodetic Science and Surveying/Ohio State University, Columbus, Ohio.

Rummel, R., Teunissen, P., 1988: Height datum definition, height datum connection and the role of the geodetic boundary value problem. Bulletin Géodésique, 62, 477-498, Springer, Berlin.

Rummel, R., Heck, B., 2001: Some critical remarks on the definition and realization of the EVRS. Veröffentlichung der Bayerischen Kommission für die Internationale Erdmessung, Nr. 61, 114-115, München.

Sacher, M., Belyashki, T., Liebsch, G., Ihde, J., 2004: Status of the UELN/EVRS Data Base and Results of the Last UELN Adjustment. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Bratislava, 2-5 June 2004, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, volume 35, pp. 172-179, Frankfurt am Main 2005.

Sacher, M., Ihde, J., Liebsch, G., Luthardt, J., 2005: Steps on the way to UELN05 and Enhancements of the web-based Geodetic Information and Service System. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Vienna, 1-4 June 2005.

Sacher, M., Ihde, J., Svensson, R., 2006: Status of UELN – Steps on the way to EVRS2007. Report on the Symposium of the IAG Subcommission for Europe (EUREF), Riga, 14-17 June 2006.

Sánchez, L., 2002: Untersuchung physikalischer Höhen in Kolumbien. Studienarbeit. Institut für Planetare Geodäsie, TU Dresden.

Sánchez, L., 2002: A reference surface for the unified height system in the northern part of South America. In: Ádám and Schwarz Eds. IAG Symposia 125: 84-89, Springer.

Sánchez, L., Martínez, W., 2002: Approach to the new vertical reference system for Colombia. In: Drewes et al. Eds. IAG Symposia 124: Pp. 27-33, Springer.

Sánchez, L., 2003: Bestimmung der Höhenreferenzfläche für Kolumbien. Diplomarbeit. Institut für Planetare Geodäsie, TU Dresden. November.

Sánchez, L., 2004: Proposed realisation of the South American Vertical System. SIRGAS Workshop, Aguascalientes, Mexico, December 2–10.

Sánchez, L (2006). Approach for the establishment of a global vertical reference system. Presented at the VI Hotine-Marussi Symposium. Wuhan, China. May 29 to Juny 02, 2006. (In press).

Sánchez, L. (2006). Strategy to establish a Global Vertical Reference System. Presented at the IAG Symposium GRF2006 (Geodetic Reference Frames 2006). Munich, Germany. October 9 to 13, 2006. (In press).

Sánchez, L.; Krügel, M. (2006). The role of the TIGA project in the unification of classical heigt systems. Presented at the IAG Symposium GRF2006 (Geodetic Reference Frames 2006). Munich, Germany. October 9 to 13, 2006. (In press).

Sánchez, L (2007). Definition and Realization of the SIRGAS Vertical Reference System within a Globally Unified Height System. IAG Symposia, Springer Verlag, Berlin, Heidelberg. V. 130: 638-645.

Yurkina, M. I., 1996: Gravity Potential at the Major Vertical Datum as Primary Geodetic Constant. Studia Geophys. et Geod., 40, 9-13, Prague.

Business Meeting of ICP1.2, 22 August 2005, on the IAG Scientific Symposium in Cairns

Ardalan, Alireza A., Safari, A.: Global Height Datum Unification: A new approach in the gravity potential space

Ardalan, Alireza A., Hashemi-Farahani, H.: New Global Estimates of the Geoid's Potential Value, Sea Surface Topography, and the Geoid based on 11 years of Topex/Poseidon Satellite Altimetry Data.

Ardalan, Alireza A. et al.: A new global ocean tide model based on orthonormal base functions and 11 years of Topex/Poseidon satellite altimetry data

Burša, M. et al.: Geopotential for Specifying Relativistic Amomic Time Scale and Global Vertical Reference System

Burša, M. et al.: Determination of Geopotential W0, Alicante AT Alicante LVD and its Connection to NAVD88, and IGN69 LVDs. Progress Report (July 2005)

Burša, M. et al.: A Global Vertical Reference Frame GVRF05

Ihde, J., Sánchez, L.: A Unified global height reference system as a basis for IGGOS

Ihde, J. et al.: Transformation Parameters between European Height Reference Frame and pan-European EVRS

Sánchez, L.: Definition and realization of the SIRGAS vertical reference system within a globally unified height system

ICP 1.2 Workshop, 11-12 April 2006 in Prague, Czech Republic

Bosch, W.: Mean Sea Surfaces - Determination and Calibration

Burša, M., Vatrt, V.: Confirmation W0 developed by SSG GGSA – difference between two developed value of W0

Drewes, H.: The role of the vertical reference system in IAG

Ihde, J.: Conventions for definition of a World Height System (WHS)

Ihde, J.: Procedures for the unification of height systems

Liebsch, G.: Information System for Height Reference Frames

Luz, R. T., de Freitas, S. R. C., Dalazoana, R., Heck, B.: Challenges for Realizing a Consistent Height System in Brazil

Mäkinen, J.: The tidal system of WHS

Sánchez, L.: Empirical determination of the global reference value W0.

Sánchez, L.: Connections of the South American height systems to a unified global vertical reference frame

Meeting of the Working Group of Theoretical Geodesy of the German Geodetic Commission (DGK) University Bonn, September 12, 2006

Becker, M.: Altimetrie und Geoid im Küstenbereich im Vergleich zu Gezeitenpegeln

Bosch, W.: Robinson Crusoe sucht Anschluss

Drewes, H.: Notwendigkeit eines vereinheitlichen Höhensystem aus der Sicht eines Globalen Geodätischen Observations-Systems (GGOS)

Flury, J., Rummel, R.: Zum Unterschied zwischen orthometrischen Höhen und Normalhöhen im Gebirge

Ihde, J.: Statusbericht zu den Arbeiten des (ICP)1.2 der IAG. Bonn

Ihde, J.: Vorschlag zur Definition und Realisierung eines Welthöhensystems

Ilk, H.: Definition eines globalen Vertikaldatums

Kusche, J.: Geoid, Meeresspiegel, Referenzsysteme - was ist vertikal -

Niemeier, W.: IKÜS-Projekt und DHHN-Erneuerung

Ruess, D.: A New Height System for Austria

Sánchez, L.: Konzept für die Definition und Realisierung eines globalen vertikalen Referenzsystem

Sudau, A.: Genauigkeit und Zuverlässigkeit von Pegelbeobachtungen