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Paper for Consideration by TSMAD

Problems arising on edges over very long distances

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Executive Summary:	In this paper some problems will be discussed which arise when edges over very long distances are used to encode areas covering the entire surface of the earth or large parts of it.
Related Documents:	S-57 edition 3.1.2. (also applying to S-100)
Related Projects:	-

Introduction / Background

We have observed problems with a data set of the AIO product. It contains a coverage object that is intended to be covering the entire surface of the earth. To achieve this, following coordinates have been used:

- 90° S 180° W
- 90° N 180° W
- 90° N 180° E
- 90° S 180° E
- 90° S 180° W

As discussed in the next sections several problems arise due to the fact that such coordinates belong to a geographic coordinate reference system (coordinates are in a spheroidic coordinate system on the surface of a spheroid).
Without loss of generality we will consider the earth as a sphere in the remaining document.

Analysis/Discussion

The following questions will be discussed now:

1. Can the entire surface of a sphere be defined by a simple polygon?
2. What is the longest distance between two consecutive vertices in a polygon?
3. What happens at the poles?

1. Can the entire surface of a sphere be defined by a simple polygon?

In the S-57 data model the boundaries of areas must be simple polygons. Denoting the points defining the polygon as vertices and the curves between such vertices as segments, the definition of a simple polygon is as follows:
A polygon ***P*** is said to be simple if the only points of the plane (here surface of the sphere) belonging to two polygon segments of ***P*** are the polygon vertices of ***P*** (adapted from Wolfram MathWorld). With other words there is no self intersection or coincidence at the polygon. As a consequence, the area of a simple polygon is always measurable and greater than 0.
Although not explicitly stated in S-57 the use of simple polygons is implied by all topological models except "Cartographic Spaghetti". Furthermore the use of coincident vertices is prohibited.

Can we cover the entire surface of a sphere by means of one single polygon?
We argue that no such polygon exists.
Each simple polygon will divide the surface into two regions: the region inside the polygon ***I*** and the region outside the polygon ***E***. It is a matter of definition on which side of the polygon is the interior or the exterior. On the sphere the sum of the areas of the two regions is exactly the area of the surface of the sphere ***A***: ***A*** = ***E*** + ***I***

If we claim: ***A*** = ***I*** then ***E*** = 0 or if we claim ***A*** = ***E*** then ***I*** = 0.
But the area of a simple polygon cannot be 0. This is a contradiction to the fact that the polygon must be simple and our original claim that no simple polygon can cover the entire surface is true.

2. What is the longest distance between two consecutive vertices in a polygon?

The geometric points between two vertices of a polygon are defined by the interpolation method. On the surface of a sphere it is usually the great circle between these two points or the loxodromic line (as defined by a clarification to S-57, see MD8, 3.8 Geometry). Please note that this will be always the shortest connection between these two points for the given interpolation method. As an example: If the coordinates of the two points are 50°N 170°W and 50°N 120°E, the connection will be crossing the 180° meridian. If the intention is that the connection should cross the 0° meridian, an intermediate point must be inserted as a vertex between the two vertices according to the interpolation method. The exact maximum distance depends on the interpolation method. For great circles the distance must be smaller than the half circumference of the sphere. For loxodromic lines the situation is slightly more complex but the difference in longitudes must be smaller than 180° in any case.

3. What happens at the poles?

The poles in a geographic coordinate reference system are in many ways interesting. They are singularities or degenerated points in the spherical coordinate system. Important is the fact that there is no one to one relation between the position and a pair of coordinates. The longitude of the pole is not to be determined and so far meaningless. The coordinate pairs shown in the introduction which at a first view show four different positions are in reality only two different positions, the north pole and the south pole. A connection between the two poles as used in the introduction is ambiguous since every meridian would be a valid connection. For the general case of a loxodromic line the pole is not a valid point. Such lines are spirals which converge to one pole but never reach it. Only loxodromic lines with an azimuth of 0° or 180° may contain the pole.
Another aspect regarding the pole is to specify a polygon that contains the pole in its interior. Such a polygon will intersect each meridian as long as the other pole is not inside, too.

Conclusions

If very large areas have to be encoded in a data product using a geographic CRS, the fact that such coordinates belong to a spheroidic coordinate system has to be taken into account. In particular attention should be paid to singularities at the poles and to connections between points that are longer than half of the circumference. In addition it should be reviewed if there is really a need to encode such big areas at all.

Recommendations

In order to achieve unambiguous definitions of long edges and large areas we recommend observing the following rules:

- The longitude difference between two vertices of an edge must be smaller than 180°
- A pole should only be part of an edge if the segments are meridians.
- There should be no direct connections between north pole and south pole.
- In order to cover the entire surface of the earth use at least two area objects or abstain completely from direct coding.

Justification and Impacts

Following the recommendations mentioned above the quality of data can be improved significantly. In particular the chance of a possible misinterpretation will be reduced. We think that an encoding bulletin would be an appropriate method to ensure that data producers will be aware about the topic.

Action Required of TSMAD

TSMAD is invited to:

a. endorse our recommendations.
b. develop and issue an encoding bulletin
c. investigate S-100 to find an appropriate section for the recommendations above.

Hamburg, February 2014